PRINCIPI MODERNIH TELEKOMUNIKACIJA
(SI2PMT)

Elektrotehnički fakultet
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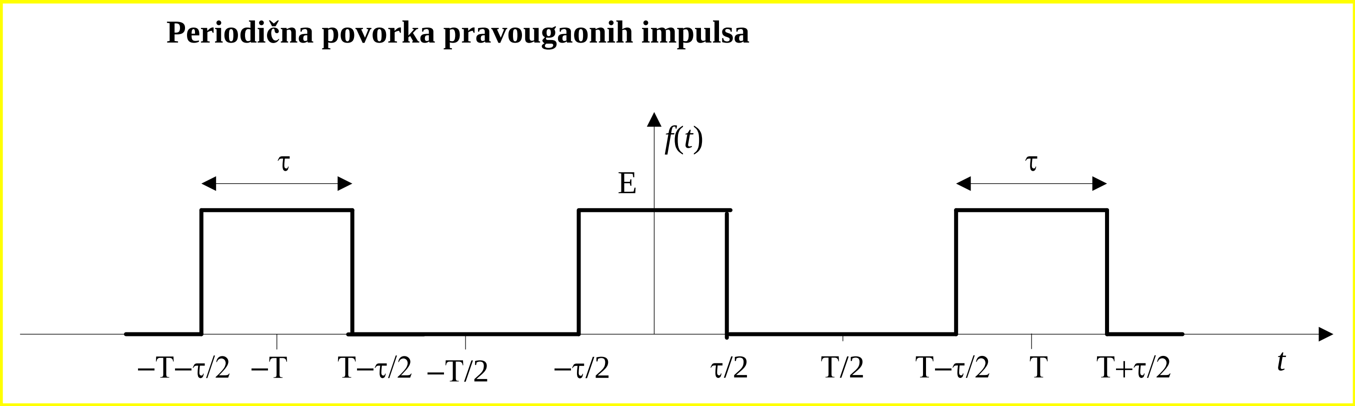
Beograd, 2012/2013.

* **Odrediti amplitudski i fazni spektar periodičnog signala $f(t)$ periode T, amplitude E i trajanja impulsa τ, koji je na intervalu jedne periode definisan sledećim izrazom**

$$f(t)=\begin{cases}0, & -T/2<t\le-\tau/2\\ E, & -\tau/2<t\le\tau/2\\ 0, & \tau/2<t\le T/2\end{cases}$$



Zadatak: Spektralna analiza periodične povorke pravougaonih impulsa (2)

* $f(t)$ je periodična funkcija, jer važi $f(t)=f(t+T)$
  * Perioda funkcije je T [s]
  * Osnovna učestanost je $\omega_0=2\pi/T$ [rad/s]
  * Osnovna frekvencija je $f_0=1/T$ [Hz]

* Funkcija $f(t)$ se može predstaviti Fourierovim redom ako zadovoljava *Dirichlet*-ov uslov

$$\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}\left|f(t)\right|dt<+\infty$$

* Fourierov red za periodičnu funkciju $f(t)$ izražen u kompleksnoj formi

$$f(t)=\sum_{n=-\infty}^{\infty}F_n e^{+jn\omega_0 t}$$

* Kompleksni koeficijent *Fourier*-ovog reda $f(t)$

$$F_n=\frac{1}{T}\int_{-T/2}^{T/2}f(t)e^{-jn\omega_0 t}dt, \qquad n=0,\pm1,\pm2,....$$

Zadatak: Spektralna analiza periodične povorke pravougaonih impulsa (3)

$$\boxed{F_n = |F_n| e^{j\theta_n}}$$

* F_n - kompleksni spektar funkcije $f(t)$
* $|F_n|$ - amplitudski spektar
* θ_n - fazni spektar

Za slučaj kada je signal *f*(*t*) periodična povorka pravougaonih impulsa, kompleksni koeficijent Fourier-ovog reda dalje se može predstaviti sa

$$F_n = |F_n| e^{j\theta_n} = \frac{1}{T} \int_{-T/2}^{T/2} f(t) e^{-jn\omega_0 t} dt = \frac{1}{T} \int_{-\tau/2}^{\tau/2} E e^{-jn\omega_0 t} dt$$

$$F_n = \frac{1}{T} \frac{1}{(-jn\omega_0)} e^{-jn\omega_0 t} \Big|_{-\tau/2}^{+\tau/2} = \frac{E\tau}{T} \frac{1}{n\omega_0 \tau/2} \frac{e^{jn\omega_0 \tau/2} - e^{-jn\omega_0 \tau/2}}{2j}$$

$$\sin(n\omega_0 \tau/2) = \frac{e^{jn\omega_0 \tau/2} - e^{-jn\omega_0 \tau/2}}{2j}$$

$$\boxed{F_n = \frac{E\tau}{T} \frac{\sin(n\omega_0 \tau/2)}{n\omega_0 \tau/2}}$$

Zadatak: Spektralna analiza periodične povorke pravougaonih impulsa (4)

Amplitudski spektar

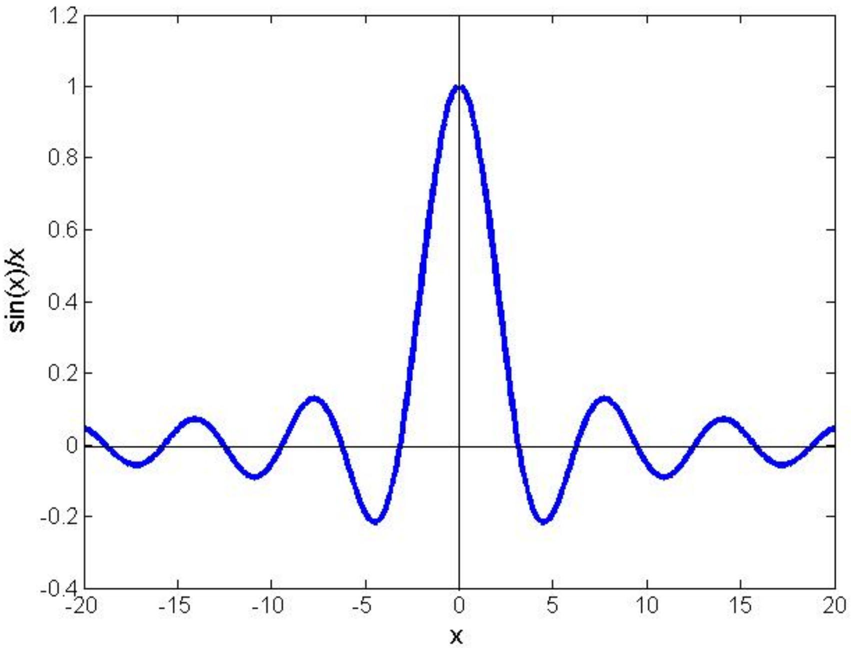
$$\left|F_n\right| = \frac{E\tau}{T}\left|\frac{\sin(n\omega_0\tau/2)}{n\omega_0\tau/2}\right|$$

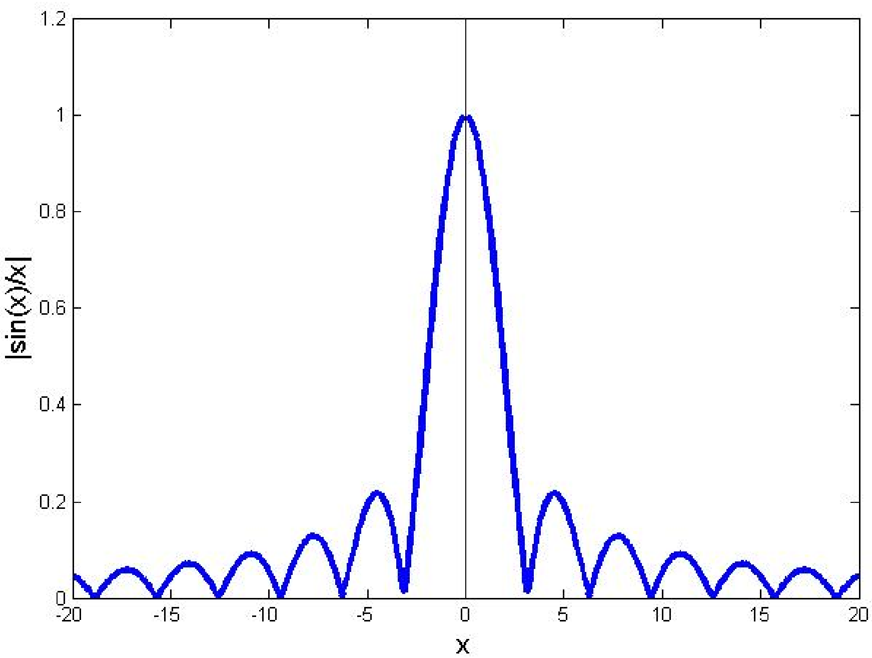
parna funkcija učestanosti

Fazni spektar

$$\theta_n = \arg(F_n) = \begin{cases} 0, \sin(n\omega_0\tau/2) \geq 0 \\ \pm\pi, \sin(n\omega_0\tau/2) < 0 \end{cases}$$

neparna f-ja učestanosti





Zadatak: Spektralna analiza periodične povorke pravougaonih impulsa (5)
nule anvelope

$$\alpha(\omega) = \frac{E\tau}{T}\left|\frac{\sin(\omega\tau/2)}{\omega\tau/2}\right|$$

Anvelopa amplitudskog spektra - pomoćna kontinualna funkcija za spektralnu analizu signala

Nule anvelope amplitudskog spektra javljaju se na učestanostima gde važi

$$\boxed{\sin(\omega_k\tau/2) = 0,\ \ \omega \neq 0}$$

$$\omega_k\tau/2 = k\pi,\ \ k = \pm1, \pm2, \ldots$$

$$\omega_k = 2k\pi/\tau,\ \ k = \pm1, \pm2, \ldots$$

***Za vrednost faktora režima (odnos trajanja impulsa i periode)** $\boxed{\dfrac{\tau}{T} = \dfrac{1}{p}}$ **nule anvelope će se nalaziti na sledećim učestanostima**

$$\omega_k = \frac{2\pi}{\tau}k = \frac{2\pi}{T}\frac{T}{\tau}k = \frac{2\pi}{T}p\cdot k = \omega_0 p\cdot k,\ \ k = \pm1, \pm2, \ldots$$

Zadatak: Spektralna analiza periodične povorke pravougaonih impulsa (6) komponente u spektru koje su jednake nuli

$$\left|F_n\right| = \frac{E}{p}\left|\frac{\sin(n\pi/p)}{n\pi/p}\right|$$

Vrednost svakog *p*-tog harmonika jednaka je 0

$F_p = F_{-p} = F_{2p} = F_{-2p} = ... = 0$

- **Primer 1:**
  - **Odnos trajanja impulsa i periode je** $\frac{\tau}{T} = \frac{1}{2}$
  - **Nule anvelope se nalaze na učestanostima**

$$\omega_k = \frac{2\pi}{\tau}k = \frac{2\pi}{T}\frac{T}{\tau}k = \frac{2\pi}{T}2\cdot k = 2\omega_0 \cdot k,\; k = \pm 1, \pm 2, \ldots$$

  - **Svaka druga komponenta u spektru jednaka je nuli !!!**

- **Primer 2:**
  - **Odnos trajanja impulsa i periode je** $\frac{\tau}{T} = \frac{1}{5}$
  - **Nule anvelope se nalaze na učestanostima**

$$\omega_k = \frac{2\pi}{\tau}k = \frac{2\pi}{T}\frac{T}{\tau}k = \frac{2\pi}{T}5\cdot k = 5\omega_0 \cdot k,\; k = \pm 1, \pm 2, \ldots$$

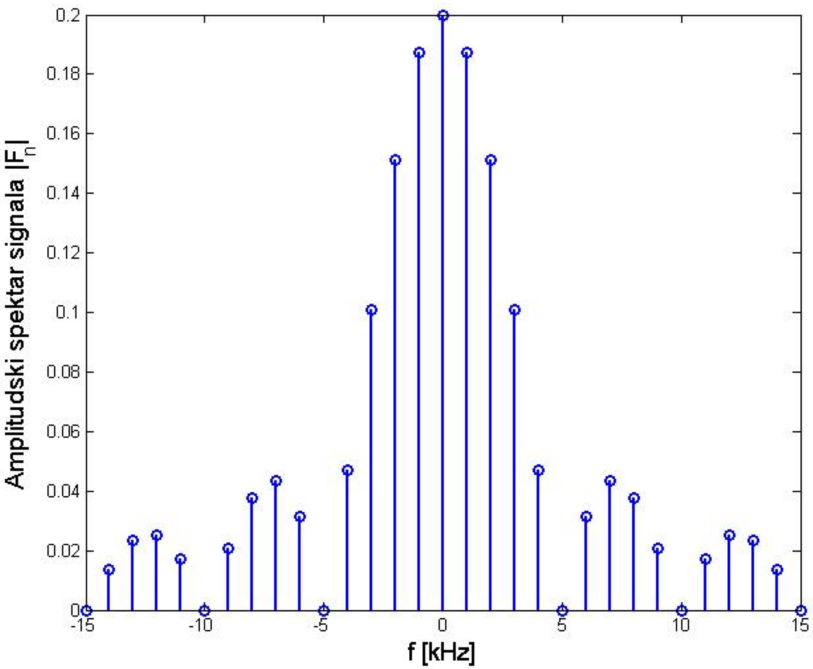
  - **Svaka peta komponenta u spektru jednaka je nuli !!!**

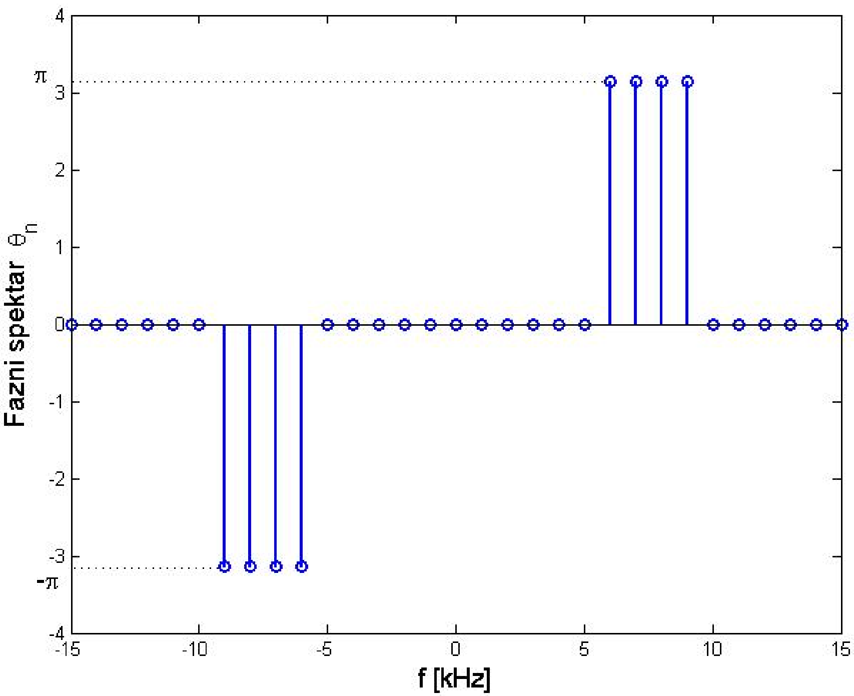
Zadatak: Spektralna analiza periodične povorke pravougaonih impulsa (7) – primer amplitudskog i faznog spektra

Perioda signala T=1ms

Osnovna frekvencija f_0=1/T=1kHz –<u>rastojanje između spektralnih komponenata</u>

Odnos trajanja impulsa i periode (faktor režima) jednak 1/5, pa je svaka peta komponenta u spektru jednaka je nuli (komponente na 5kHz, 10kHz, 15kHz, …)





Zadatak: Spektralna analiza periodične povorke pravougaonih impulsa (8) – spektar snage

Spektar snage

$$S_{11}(n\omega_0) = |F_n|^2 = \left(\frac{E\tau}{T}\right)^2 \left(\frac{\sin(n\omega_0\tau/2)}{n\omega_0\tau/2}\right)^2 = \left(\frac{E}{p}\right)^2 \left(\frac{\sin(n\pi/p)}{n\pi/p}\right)^2$$

Srednja snaga signala *f*(*t*)

$$P_S = \frac{1}{T}\int_{-T/2}^{T/2} f^2(t)\,dt = \frac{1}{T}\int_{-\tau/2}^{\tau/2} E^2 dt = E^2\frac{\tau}{T} = \frac{E^2}{p}$$

<u>Parsevalova teorema</u>:

Srednja snaga složenog periodičnog signala jednaka je zbiru srednjih snaga svih njegovih harmonika

$$P_S = \frac{1}{T}\int_{-T/2}^{T/2} f^2(t)\,dt = \sum_{n=-\infty}^{\infty} |F_n|^2 = |F_0|^2 + 2\sum_{n=1}^{\infty} |F_n|^2$$

Zadatak: Spektralna analiza periodične povorke pravougaonih impulsa (9) – spektar snage

Snaga signala *f*(*t*) koja se nalazi u prvih *N* harmonika

$$P_{h_N} = \sum_{n=-N}^{N} |F_n|^2 = |F_0|^2 + 2\sum_{n=1}^{N} |F_n|^2 = \left(\frac{E}{p}\right)^2 + 2\sum_{n=1}^{N}\left(\frac{E}{p}\right)^2 \left(\frac{\sin(n\pi/p)}{n\pi/p}\right)^2$$

Odnos snage signala *f*(*t*) koja se nalazi u prvih *N* harmonika i ukupne snage signala

$$\frac{P_{h_N}}{P_S} = \frac{\left(\frac{E}{p}\right)^2 + 2\sum_{n=1}^{N}\left(\frac{E}{p}\right)^2 \left(\frac{\sin(n\pi/p)}{n\pi/p}\right)^2}{\frac{E^2}{p}} = \frac{1}{p}\left[1 + 2\sum_{n=1}^{N}\left(\frac{\sin(n\pi/p)}{n\pi/p}\right)^2\right]$$

Primer: periodična povorka pravougaonih impulsa, faktor režima 1/2

$$\boxed{\left|F_n\right| = \frac{E}{2}\left|\frac{\sin(n\pi/2)}{n\pi/2}\right|}$$

$$\left|F_0\right| = \frac{E}{2} \qquad \left|F_1\right| = \left|F_{-1}\right| = \frac{E}{2}\left|\frac{\sin(\pi/2)}{\pi/2}\right| = \frac{E}{\pi}$$

$$\left|F_2\right| = \left|F_{-2}\right| = 0 \qquad \left|F_3\right| = \left|F_{-3}\right| = \frac{E}{2}\left|\frac{\sin(3\pi/2)}{3\pi/2}\right| = \frac{E}{3\pi}$$

$$\left|F_4\right| = \left|F_{-4}\right| = 0 \qquad \left|F_5\right| = \left|F_{-5}\right| = \frac{E}{2}\left|\frac{\sin(5\pi/2)}{5\pi/2}\right| = \frac{E}{5\pi}$$

Srednja snaga signala *f*(*t*)

$$P_S = \frac{1}{T}\int_{-T/2}^{T/2} f^2(t)\,dt = E^2\frac{\tau}{T} = \frac{E^2}{2}$$

Snaga signala *f*(*t*) koja se nalazi u prvih *N* harmonika

$$P_{h_N} = \sum_{n=-N}^{N}\left|F_n\right|^2 = \left|F_0\right|^2 + 2\sum_{n=1}^{N}\left|F_n\right|^2 = \left(\frac{E}{2}\right)^2\left[1 + 2\sum_{n=1}^{N}\left(\frac{\sin(n\pi/2)}{n\pi/2}\right)^2\right]$$

Primer: periodična povorka pravougaonih impulsa, faktor režima 1/2

Snaga signala *f*(*t*) koja se nalazi u nultom (jednosmerna komponenta) i prvom harmoniku

$$P_{h_1}=\left|F_{-1}\right|^2+\left|F_0\right|^2+\left|F_1\right|^2=\left(\frac{E}{2}\right)^2\left[1+2\left(\frac{\sin(\pi/2)}{\pi/2}\right)^2\right]=E^2\left[\left(\frac{1}{2}\right)^2+2\left(\frac{1}{\pi}\right)^2\right]=0.4526E^2$$

Odnos snage signala u nultom i prvom harmoniku i srednje snage signala (P_s $=E^2/2$) iznosi

$$\frac{P_{h_1}}{P_S}=\frac{0.4526E^2}{0.5E^2}=0.9053=90.53\%$$

Kada je faktor režima jednak ½, drugi harmonik je jednak nuli, pa je snaga signala *f*(*t*) koja se nalazi u nultom i prva dva harmonika takođe

$$P_{h_2}=\underbrace{\left|F_{-2}\right|^2}_{0}+\left|F_{-1}\right|^2+\left|F_0\right|^2+\left|F_1\right|^2+\underbrace{\left|F_2\right|^2}_{0}=\left|F_{-1}\right|^2+\left|F_0\right|^2+\left|F_1\right|^2=0.4526E^2$$

Odnos snage signala u nultom i prvom harmoniku i srednje snage signala (P_s $=E^2/2$) iznosi

$$\frac{P_{h_2}}{P_S}=\frac{0.4526E^2}{0.5E^2}=0.9053=90.53\%$$

Primer: periodična povorka pravougaonih impulsa, faktor režima 1/2

Snaga signala *f*(*t*) koja se nalazi u prva tri harmonika jednaka je snazi koja se nalazi u prva četiri harmonika (svi parni harmonici jednaki su nuli, pa je i četvrti harmonik jednak nuli!)

$$P_{h_3} = P_{h_4} = \left(\frac{E}{2}\right)^2 \left[1 + 2\left(\frac{\sin(\pi/2)}{\pi/2}\right)^2 + 2\left(\frac{\sin(2\pi/2)}{2\pi/2}\right)^2 + 2\left(\frac{\sin(3\pi/2)}{3\pi/2}\right)^2 + 2\left(\frac{\sin(4\pi/2)}{4\pi/2}\right)^2\right] =$$

$$= E^2\left[\left(\frac{1}{2}\right)^2 + 2\left(\frac{1}{\pi}\right)^2 + 2\left(\frac{1}{3\pi}\right)^2\right] = 0.4752E^2$$

Odnos snage signala u prva tri (odnosno prva četiri harmonika) i srednje snage signala iznosi

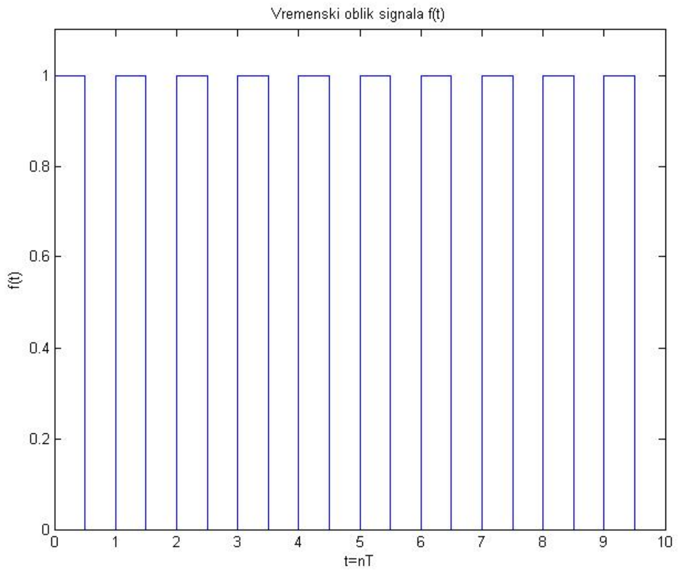
$$\frac{P_{h_3}}{P_S} = \frac{P_{h_4}}{P_S} = \frac{0.4752E^2}{0.5E^2} = 0.9503 = 95.3\%$$

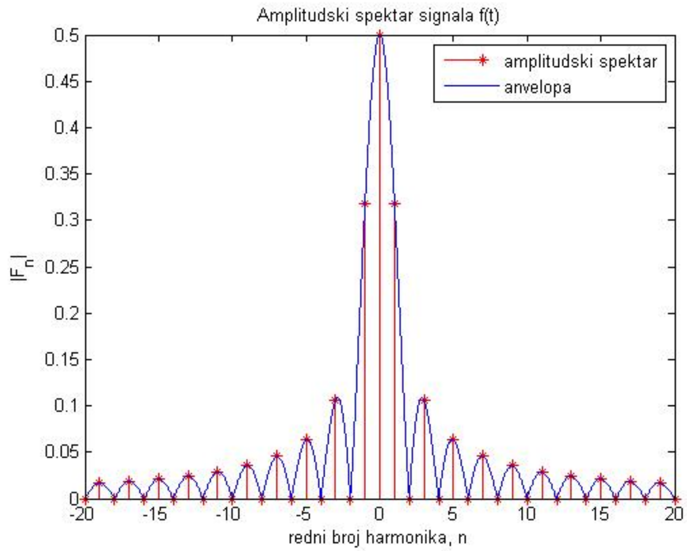
Snaga signala *f*(*t*) koja se nalazi u prvih pet (šest) harmonika jednaka je

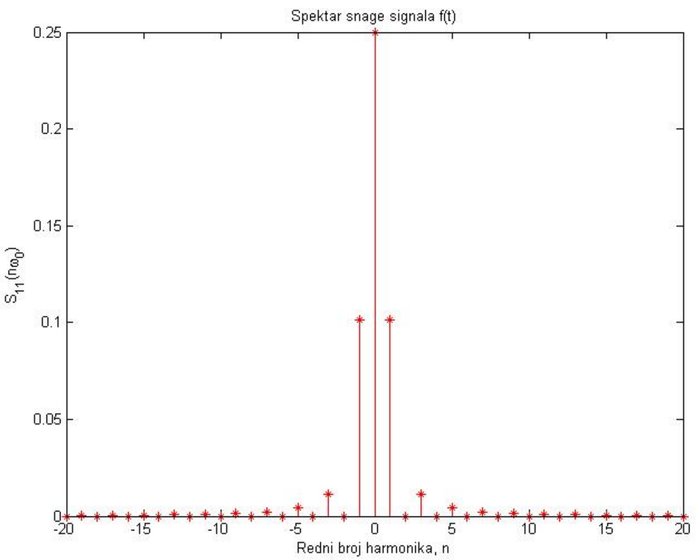
$$P_{h_5} = P_{h_6} = \left(\frac{E}{2}\right)^2 \left[1 + 2\left(\frac{\sin(\pi/2)}{\pi/2}\right)^2 + 2\left(\frac{\sin(3\pi/2)}{3\pi/2}\right)^2 + 2\left(\frac{\sin(5\pi/2)}{5\pi/2}\right)^2\right] =$$

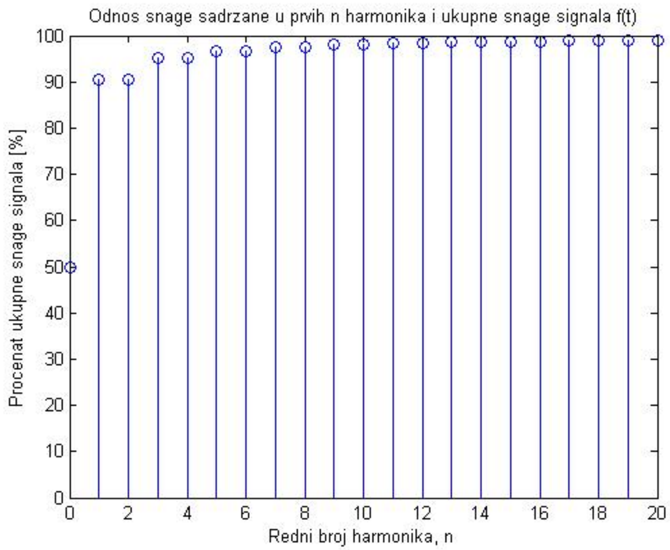
$$= E^2\left[\left(\frac{1}{2}\right)^2 + 2\left(\frac{1}{\pi}\right)^2 + 2\left(\frac{1}{5\pi}\right)^2\right] = 0.4833E^2 \Rightarrow \frac{P_{h_5}}{P_S} = \frac{P_{h_6}}{P_S} = \frac{0.4833E^2}{0.5E^2} = 0.9665 = 96.65\%$$

Primer: periodična povorka pravougaonih impulsa, faktor režima 1/2









Primer: periodična povorka pravougaonih impulsa, faktor režima 1/4

$$\boxed{\left|F_n\right| = \frac{E}{4}\left|\frac{\sin(n\pi/4)}{n\pi/4}\right|}$$

$$\left|F_0\right| = \frac{E}{4}$$

$$\left|F_1\right| = \left|F_{-1}\right| = \frac{E}{4}\left|\frac{\sin(\pi/4)}{\pi/4}\right| = \frac{E}{\pi}\frac{\sqrt{2}}{2}$$

$$\left|F_2\right| = \left|F_{-2}\right| = \frac{E}{4}\left|\frac{\sin(2\pi/4)}{2\pi/4}\right| = \frac{E}{2\pi}$$

$$\left|F_3\right| = \left|F_{-3}\right| = \frac{E}{4}\left|\frac{\sin(3\pi/4)}{3\pi/4}\right| = \frac{E}{3\pi}\frac{\sqrt{2}}{2}$$

$$\left|F_4\right| = \left|F_{-4}\right| = \frac{E}{4}\left|\frac{\sin(4\pi/4)}{4\pi/4}\right| = 0$$

$$\left|F_5\right| = \left|F_{-5}\right| = \frac{E}{4}\left|\frac{\sin(5\pi/4)}{5\pi/4}\right| = \frac{E}{5\pi}\frac{\sqrt{2}}{2}$$

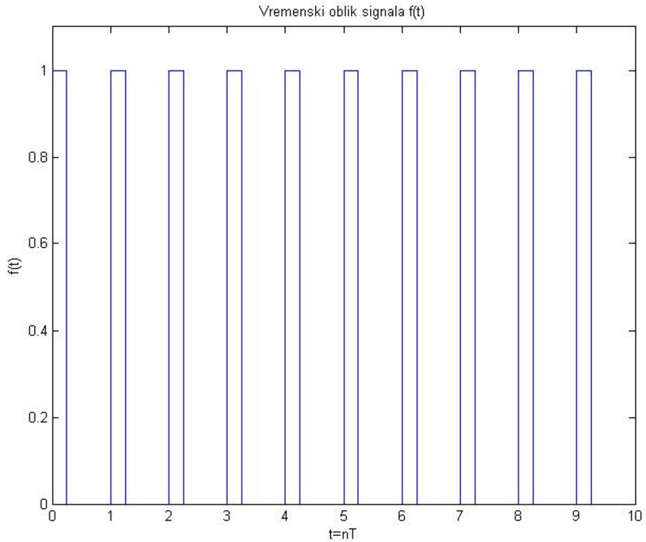
Srednja snaga signala *f*(*t*)

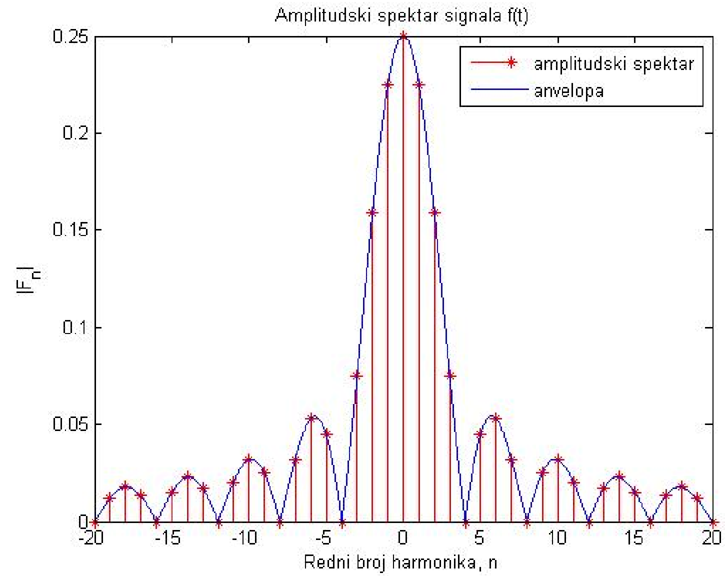
$$P_S = \frac{1}{T}\int_{-T/2}^{T/2} f^2(t)\,dt = E^2\frac{\tau}{T} = \frac{E^2}{4}$$

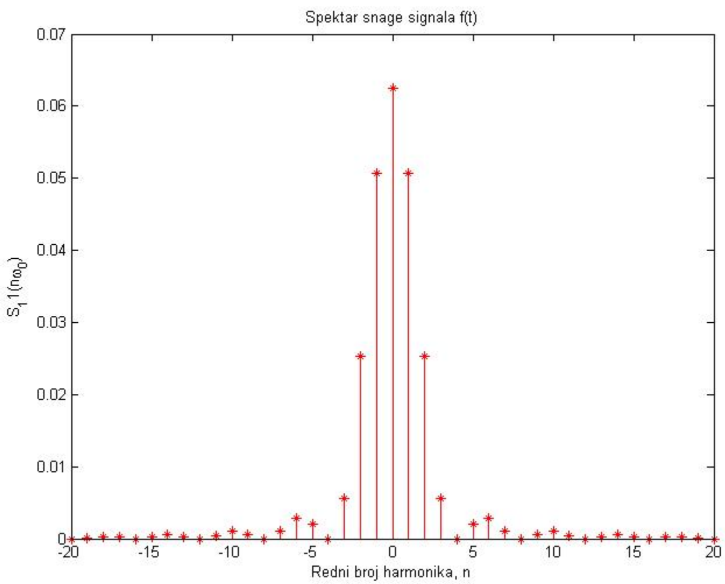
Snaga signala *f*(*t*) koja se nalazi u prvih *N* harmonika

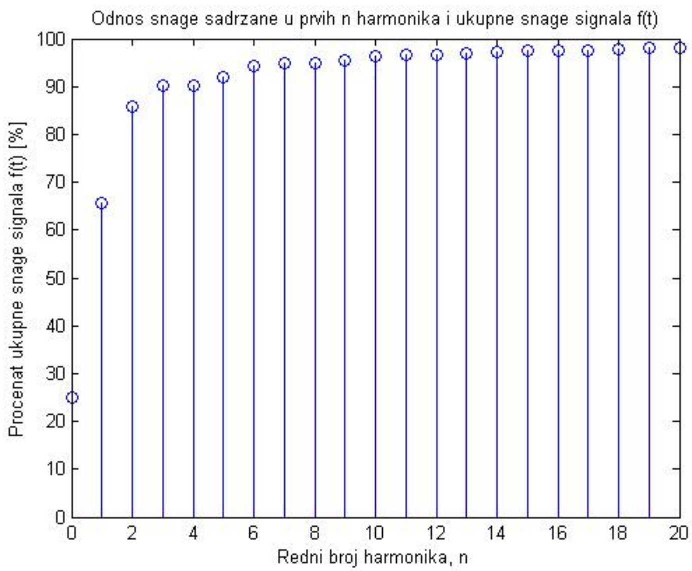
$$P_{h_N} = \sum_{n=-N}^{N}\left|F_n\right|^2 = \left|F_0\right|^2 + 2\sum_{n=1}^{N}\left|F_n\right|^2 = \left(\frac{E}{4}\right)^2\left[1 + 2\sum_{n=1}^{N}\left(\frac{\sin(n\pi/4)}{n\pi/4}\right)^2\right]$$

Primer: periodična povorka pravougaonih impulsa, faktor režima 1/4









Primer: periodična povorka pravougaonih impulsa, faktor režima 1/8

*Dat je periodičan signal čija je perioda T=2ms a trajanje impulsa τ=250μs.

a) Odrediti faktor režima
b) Odrediti osnovnu frekvenciju signala
c) Amplitudski spektar i spektar snage signala
d) Koliki procenat srednje snage signala se nalazi na učestanostima nižim od 1.8kHz
e) U koliko prvih *N* harmonika je sadržano više od 50% snage signala *f*(*t*)

a) Faktor režima $\frac{\tau}{T}=\frac{250\mu s}{2ms}=\frac{1}{8}$

b) Osnovna frekvencija signala $f_0=\frac{1}{T}=\frac{1}{2ms}=0.5kHz$

Rastojanje između spektralnih komponenti jednako je osnovnoj učestanosti signala, odnosno jednako je 0.5kHz=500Hz

c) Amplitudski spektar signala

$$\left|F_n\right|=\frac{E}{8}\left|\frac{\sin(n\pi/8)}{n\pi/8}\right|$$

Spektar snage

$$S_{11}\left(n\omega_0\right)=\left|F_n\right|^2=\frac{E^2}{8^2}\left|\frac{\sin(n\pi/8)}{n\pi/8}\right|^2$$

Primer: periodična povorka pravougaonih impulsa, faktor režima 1/8

Srednja snaga signala *f*(*t*)

$$P_S = \frac{1}{T}\int_{-T/2}^{T/2} f^2(t)\,dt = E^2\frac{\tau}{T} = \frac{E^2}{8}$$

d) Na učestanostima nižim od 1.8kHz nalaze se jednosmerna komponenta i prva tri harmonika u spektru signala (na f_0=0.5kHz, 2f_0=1kHz, 3f_0=1.5kHz)

$$P_{h3} = \left(\frac{E}{8}\right)^2\left[1+2\left(\frac{\sin(\pi/8)}{\pi/8}\right)^2+2\left(\frac{\sin(2\pi/8)}{2\pi/8}\right)^2+2\left(\frac{\sin(3\pi/8)}{3\pi/8}\right)^2\right]=0.0899E^2 \Rightarrow$$

$$\frac{P_{h3}}{P_S}=\frac{0.0899E^2}{0.125E^2}=71.92\%$$

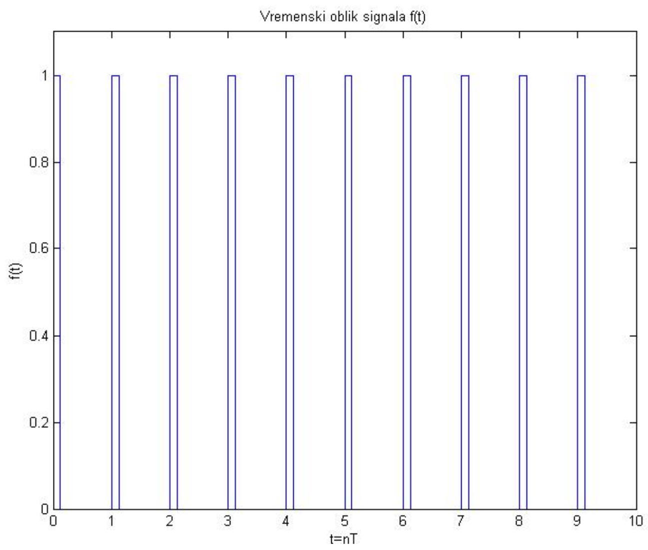
e) U jednosmernoj komponenti i u prvom harmoniku (f_0=0.5kHz) sadržano je 36.24% srednje snage signala *f*(*t*)

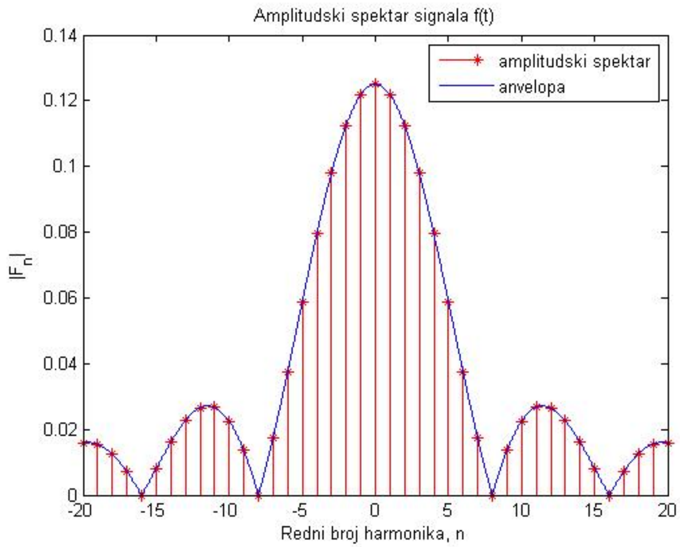
$$P_{h1} = \left(\frac{E}{8}\right)^2\left[1+2\left(\frac{\sin(\pi/8)}{\pi/8}\right)^2\right]=0.0453E^2 \Rightarrow \frac{P_{h1}}{P_S}=\frac{0.0453E^2}{0.125E^2}=36.24\%$$

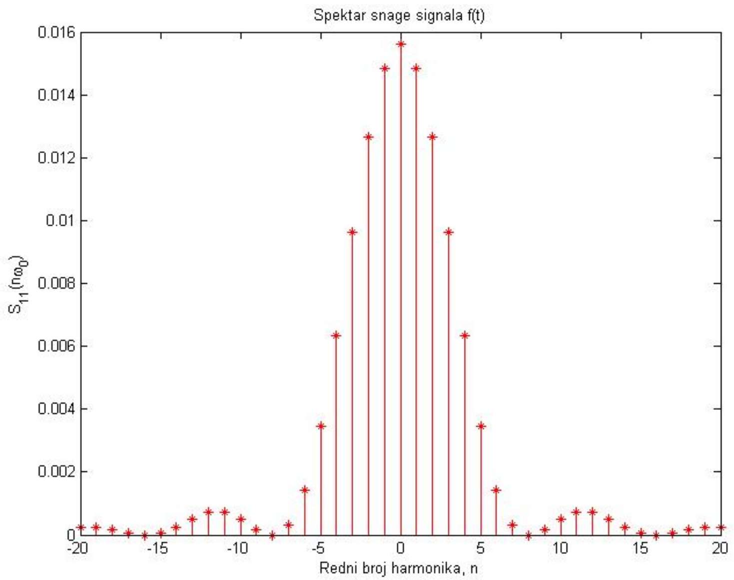
$$P_{h2} = \left(\frac{E}{8}\right)^2\left[1+2\left(\frac{\sin(\pi/8)}{\pi/8}\right)^2+2\left(\frac{\sin(2\pi/8)}{2\pi/8}\right)^2\right]=0.0706E^2 \Rightarrow \frac{P_{h1}}{P_S}=\frac{0.0706E^2}{0.125E^2}=56.5\%$$

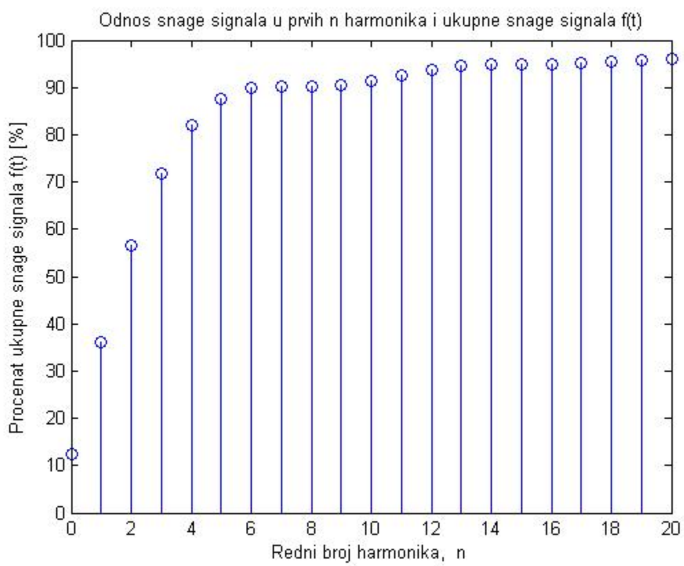
U jednosmernoj komponenti i u prva dva harmonika (na f_0=0.5kHz, 2f_0=1kHz) sadržano je više od 50% srednje snage signala *f*(*t*)

Primer: periodična povorka pravougaonih impulsa, faktor režima 1/8





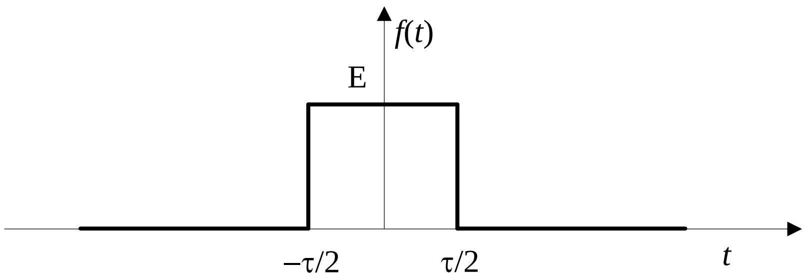




Zadatak: Spektralna analiza usamljenog pravougaonog impulsa (1)

* **Odrediti spektralnu gustinu amplituda i faza signala *f*(*t*) datog sa**

$$f(t)=\begin{cases}0, & -\infty < t \le -\tau/2 \\ E, & -\tau/2 < t \le +\tau/2 \\ 0, & +\tau/2 < t \le +\infty\end{cases}$$



Fourier-ova transformacija usamljenog pravougaonog impulsa *f*(*t*)

$$F(j\omega)=\left|F(j\omega)\right|e^{+j\theta(\omega)}=\int_{-\infty}^{+\infty} f(t)e^{-j\omega t}dt=\int_{-\tau/2}^{+\tau/2} E\underbrace{e^{-j\omega t}}_{\cos(\omega t)-j\sin(\omega t)}dt=$$

$$=\int_{-\tau/2}^{+\tau/2} E\underbrace{e^{-j\omega t}}_{\cos(\omega t)-j\sin(\omega t)}dt=E\int_{-\tau/2}^{+\tau/2}\cos(\omega t)dt-\underbrace{jE\int_{-\tau/2}^{+\tau/2}\sin(\omega t)dt}_{\mathbf{0}}=$$

$$=E\int_{-\tau/2}^{+\tau/2}\cos(\omega t)dt=2E\int_{0}^{+\tau/2}\cos(\omega t)dt=2E\frac{\sin(\omega\tau/2)}{\omega}=E\tau\frac{\sin(\omega\tau/2)}{\omega\tau/2}$$

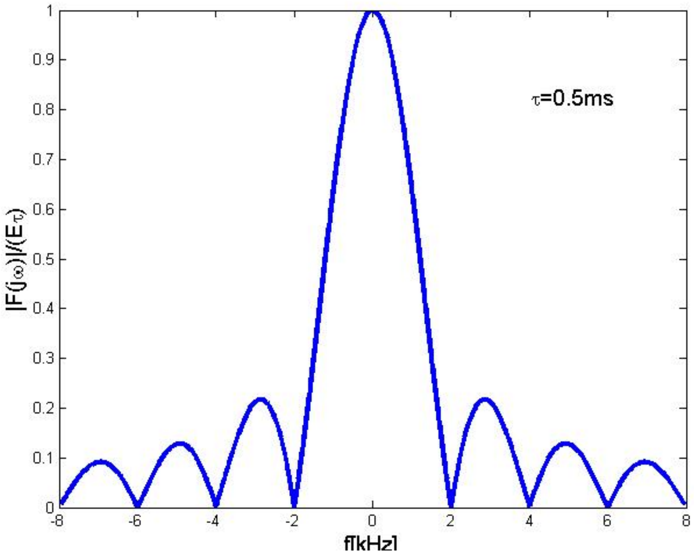
$$F(j\omega)=E\tau\frac{\sin(\omega\tau/2)}{\omega\tau/2}$$

Zadatak: Spektralna analiza usamljenog pravougaonog impulsa (2)

*Spektralna gustina amplituda

$$\left|F\left(j\omega\right)\right| = E\tau\left|\frac{\sin\left(\omega\tau/2\right)}{\omega\tau/2}\right|$$

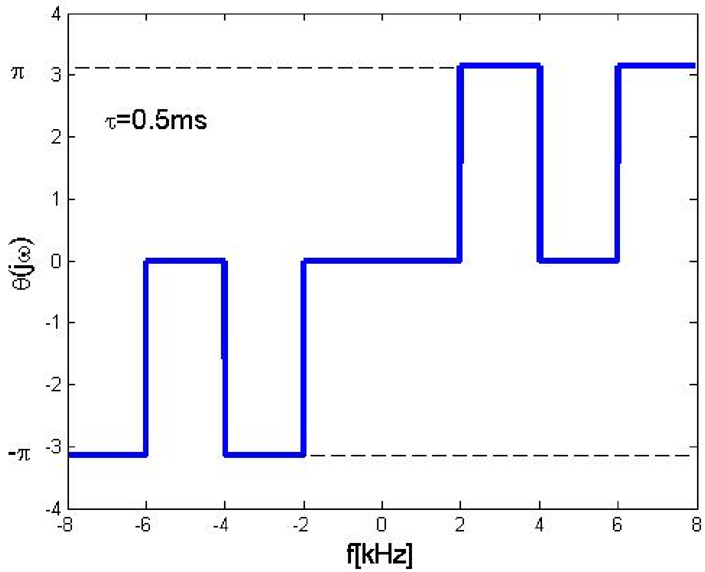
parna funkcija učestanosti



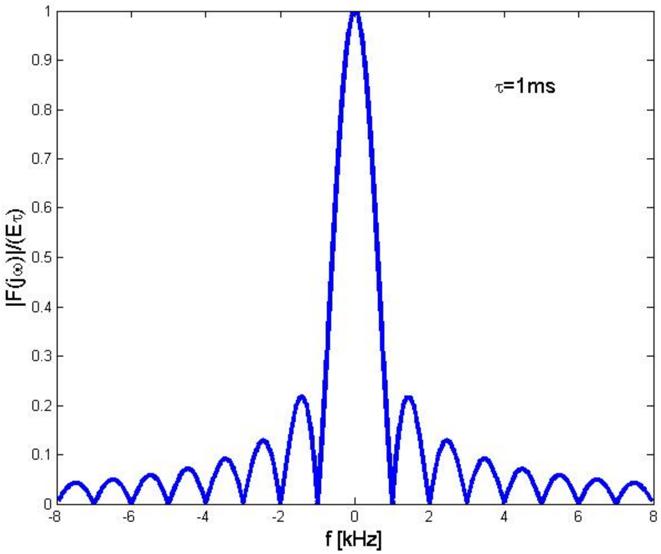
*Spektralna gustina faza

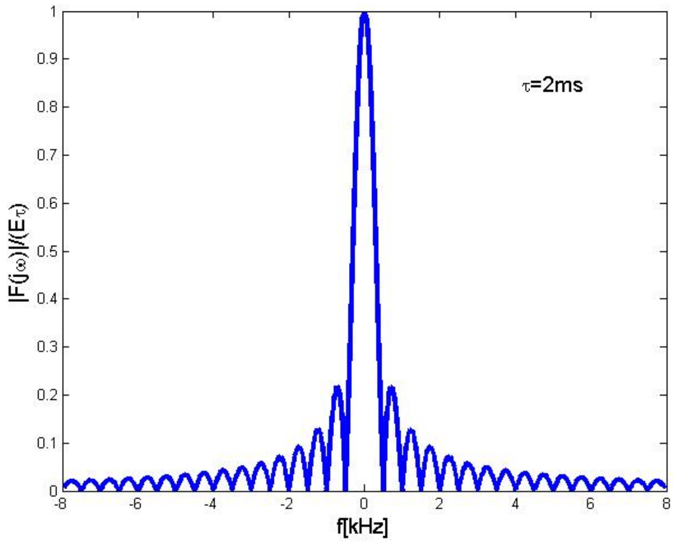
$$\theta_n = \arg(F_n) = \begin{cases} 0, \sin(\omega\tau/2) \geq 0 \\ \pm\pi, \sin(\omega\tau/2) < 0 \end{cases}$$

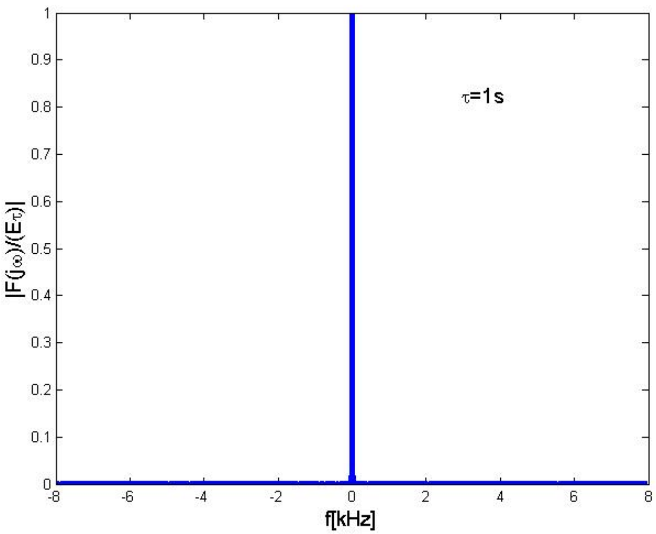
neparna funkcija učestanosti

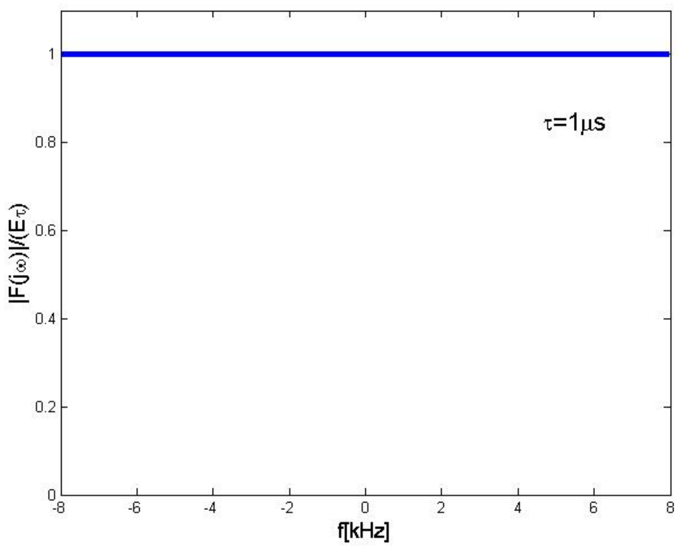


Zadatak: Spektralna analiza usamljenog pravougaonog impulsa (3) – razne vrednosti trajanja impulsa τ



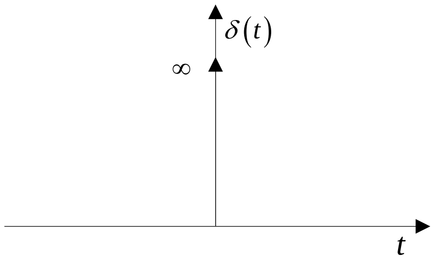


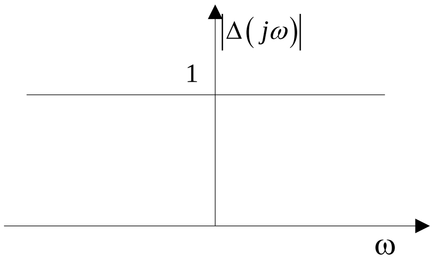




Zadatak: Spektralna analiza usamljenog pravougaonog impulsa (4) – granični slučajevi

*Dirakov impuls



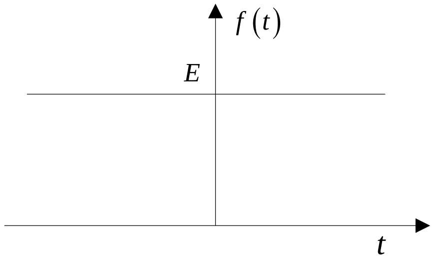


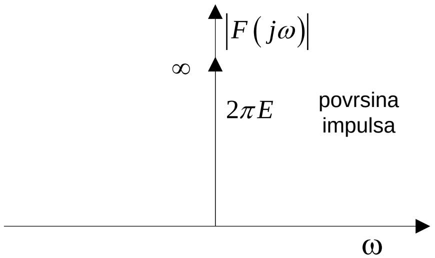
$$\tau \to 0,\ E\tau = 1,\ \delta(t) = \lim_{\tau\to 0} f(t)$$

$$\Delta(j\omega) = \int_{-\infty}^{+\infty} \delta(t) e^{-j\omega t} dt = 1$$

$$\delta(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \Delta(j\omega) e^{+j\omega t} d\omega = \frac{1}{2\pi}\int_{-\infty}^{+\infty} 1\cdot e^{+j\omega t} d\omega = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \cos(\omega t) d\omega + j\frac{1}{2\pi}\int_{-\infty}^{+\infty} \sin(\omega t) d\omega = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \cos(\omega t) d\omega$$

*Funkcija konstante





$$f(t) = E$$

$$F(j\omega) = \int_{-\infty}^{+\infty} E e^{-j\omega t} dt = \int_{-\infty}^{+\infty} E\cos(\omega t) d\omega - j\int_{-\infty}^{+\infty} E\sin(\omega t) d\omega = E\int_{-\infty}^{+\infty} \cos(\omega t) d\omega$$

$$F(j\omega) = 2\pi E\delta(\omega)$$

Literatura

[1] Dukić M., *Principi telekomunikacija*, Akademska misao, 2008, Beograd.

[2] Haykin S., *Communication Systems*, John Wiley&Sons, *Inc*., 1998, New York.

[3] Dukić M., Marković G., Vujić D., *Principi telekomunikacija – Zbornik rešenih zadataka*, Akademska misao, 2009, Beograd.

[4] Praktikum za laboratorijske vežbe.

***Zadaci urađeni po uzoru na zadatke 1.1-1.4, 1.8 iz [3].**