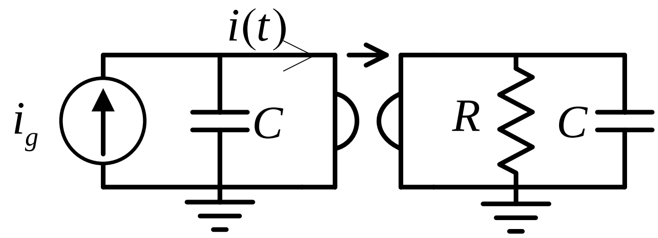
Zadatak 1



Slika 1

U kolu na slici 1, poznatih parametara C i $R = r = \sqrt{\frac{L}{C}}$, deluje strujni generator $i_g(t) = I(h(t) - h(t-T))$, $T = RC$. Rešavanjem kola u vremenskom domenu odrediti:

[70] a) Indicionu funkciju za struju $i_1(t)$.

[30] b) Struju $i_L(t)$, za $t \geq 0$.

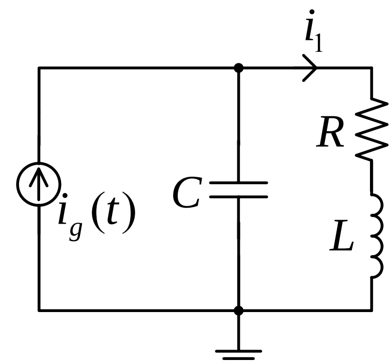
1 Zadatak Slika 1 a

$$(D^2 + 2\alpha D + 4\alpha^2) i_L = 4\alpha^2 \cdot i_g; \qquad \alpha = \frac{1}{2RC}$$

a)

$$f(t) = \{1 - e^{-\alpha t}[\cos(\sqrt{3}\alpha t) + \frac{1}{\sqrt{3}}\sin(\sqrt{3}\alpha t)]\}h(t)$$

b) $i_L(t) = I \cdot f(t) - I \cdot f(t-T)$



Slika 1a

Zadatak 2

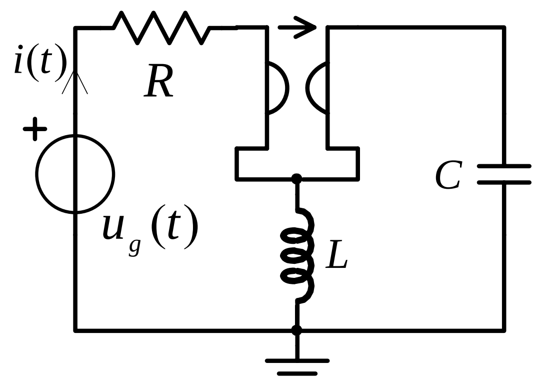
Parametri L, C i $R = r = \sqrt{\frac{L}{C}}$, kola sa slike 2, su poznati.

Odrediti:

[40] **a)** Ulaznu admitansu mreže vezane za krajeve generatora, $\underline{Y}_{ul}(\underline{s}) = I / \underline{U}_g$.

[40] **b)** Trenutnu vrednost struje generatora $i(t)$ za $u_g(t) = 5U\cos\omega_1 t + 13U\cos 2\omega_1 t + U\cos 3\omega_1 t$, pri čemu je $\omega_1 = 1/(3\sqrt{LC})$.

[20] **c)** Efektivne vrednosti napona i struje generatora.



Slika 2.

2.a) $$\underline{Y}_{ul} = \frac{1 + \underline{s}^2 R^2 C^2}{R(\underline{s}^2 R^2 C^2 + 2\underline{s}RC + 1)};$$

b) $$\underline{Y}_{ul}^{(n)} = \frac{\omega_0^2 - n^2\omega_1^2}{R(\omega_0 + jn\omega_1)^2}, \quad \omega_0 = \frac{1}{RC} = 3\omega_1$$

$$\underline{I}^{(n)} = \underline{Y}_{ul}^{(n)} \cdot \underline{U}_g^{(n)},$$

$$\underline{I}^{(1)} = \frac{4U}{\sqrt{2}R} e^{-j\arg(\frac{3}{4})} \to i^{(1)}(t) = \frac{4U}{R}\cos(\omega_1 t - \mathrm{arctg}\frac{3}{4})$$

$$\underline{I}^{(2)} = \frac{5U}{\sqrt{2}R} e^{-j\arg(\frac{12}{5})} \to i^{(2)}(t) = \frac{5U}{R}\cos(2\omega_1 t - \arg\frac{12}{5})$$

$\underline{I}^{(3)} = 0 \to i^{(3)}(t) = 0$ pa je

$$i(t) = \frac{4U}{R}\cos(\omega_1 t - \arg\frac{3}{4}) + \frac{5U}{R}\cos(2\omega_1 t - \arg\frac{12}{5})$$

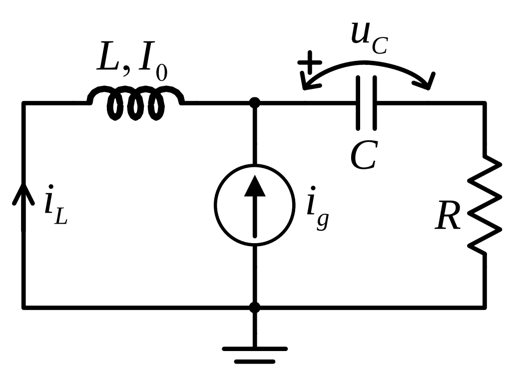
c) $$U_{eff} = U\sqrt{\frac{195}{2}}, \qquad I = \frac{U}{R}\sqrt{\frac{41}{2}}.$$

Zadatak 1

U kolu poznatih parametara (slika 1) R, L i $C = \frac{4L}{5R^2}$, deluje strujni generator $i_g(t) = \frac{I \cdot t}{RC} h(t)$. Struja kalema u trenutku $t = 0$ iznosi $i_L(0^-) = I_0$. Odrediti:

a)[30] diferencijalnu jednačinu odziva napona kondenzatora $u_C(t)$,

b)[50] i rešavanjem kola u vremenskom domenu, napon kondenzatora $u_C(t)$ za $t \geq 0$.



Slika 1

a) $(D^2+2\alpha D+\frac{1}{LC})\cdot u_C=\frac{1}{C}\cdot Di_g;\quad \alpha=\frac{R}{2L}$

b) $(D^2+2\alpha D+\frac{1}{LC})\cdot u_C=\frac{I}{RC^2},\ t\ge 0.$

$\underline{s}_{1,2}=-\alpha\pm j\omega_1,\ \omega_1=2\alpha,$

$u_c(t)=\left(A\cdot\cos\omega_1 t+B\cdot\sin\omega_1 t\right)\cdot e^{-\alpha t}+u_{cP}(t)$

$u_{cP}(t)=const=\frac{5}{4}RI,\ i_g(0^-)=0,$

$$Du_c(0^-)=\frac{i_C(0^-)}{C}=\frac{1}{C}\left[i_L(0^-)+i_g(0^-)\right]=\frac{I_0}{C},$$

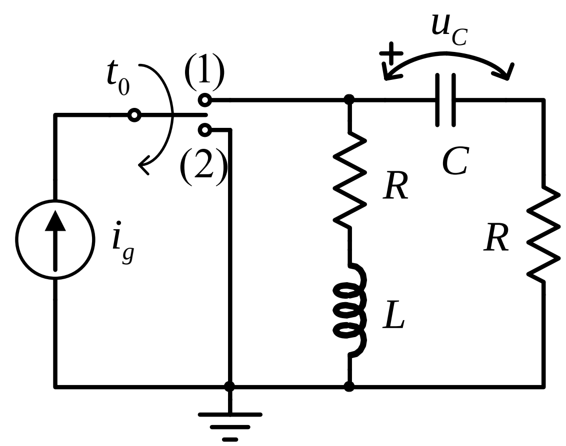
$$Du_c(0^-)=-\alpha A+2\alpha B=\frac{I_0}{C}$$

$$A=-\frac{5}{4}RI,\ B=\frac{5}{4}R\left(\frac{I}{2}-I_0\right),$$

$$u_c(t)=\frac{5}{4}RI\left\{1-\left[\cos\omega_1 t+\left(\frac{1}{2}-\frac{I_0}{I}\right)\cdot\sin\omega_1 t\right]\cdot e^{-\alpha t}\right\}\cdot h(t).$$

Zadatak 3

[20+40+40] U kolu poznatih parametara (slika 3) R, L i $C=\frac{L}{5R^2}$, deluje generator konstantne struje $i_g(t)=I$. Prekidač se nalazi u položaju (1) i kolo je u ustaljenom režimu. U trenutku $t=t_0$ prekidač se prebaci u položaj (2). Primenom *Laplace*–ove transformacije odrediti napon kondenzatora $u_C(t)$ za $t\ge 0$.



Slika 3

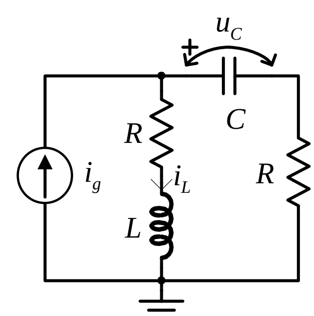
Za $0\le t<t_0$ Slika 3a Samo ispit

$$\underline{U}_C=\frac{\frac{\underline{s}}{C}-\frac{\alpha}{C}}{\underline{s}^2+2\alpha\underline{s}+\omega_0^2}I_g=\frac{1}{C}\frac{\underline{s}-\alpha}{\left(\underline{s}+\alpha\right)^2+\omega_1^2}\cdot\frac{I}{\underline{s}}=$$

$$=\frac{I}{\omega_1 C}\frac{\omega_1}{\left(\underline{s}+\alpha\right)^2+\omega_1^2}+\frac{1}{C}\frac{\alpha}{\left(\underline{s}+\alpha\right)^2+\omega_1^2}\cdot\frac{I}{\underline{s}}=$$

$$=I\frac{1}{\omega_1 C}\frac{\omega_1}{\left(\underline{s}+\alpha\right)^2+\omega_1^2}+\frac{RI}{\underline{s}}-RI\frac{\underline{s}+\alpha}{\left(\underline{s}+\alpha\right)^2+\omega_1^2}-\frac{RI}{2}\frac{\omega_1}{\left(\underline{s}+\alpha\right)^2+\omega_1^2}$$

$$\xrightarrow{L-1}u_C(t)=RI\left\{1+e^{-\alpha t}\left[2\sin(\omega_1 t)-\cos(\omega_1 t)\right]\right\}, 0<t>t_0$$



Slika 3a

Ispit i kolokvijum:

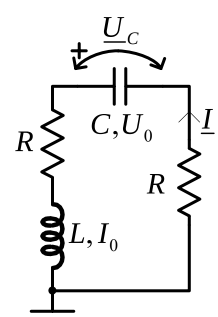
Za $t=t_0,\ i_L\left(t_0^-\right)=I=-I_0,\ u_C\left(t_0^-\right)=RI=U_0.$

Za $t\ge t_0,\ \tau=t-t_0.$ Slika 3b

$$\left.\begin{cases}2R\underline{I}+L\underline{s}\underline{I}-LI_0+\underline{U}_C=0,\\ \underline{I}=C\underline{s}\underline{U}_C-CU_0\end{cases}\right\}\Rightarrow\underline{I}=\frac{\underline{s}I_0-\frac{U_0}{L}}{\underline{s}^2+2\frac{R}{L}\underline{s}+\frac{1}{LC}}$$

$$\begin{cases}\underline{I}=I_0\frac{\underline{s}+\alpha}{\left(\underline{s}+\alpha\right)^2+\omega_1^2}\underbrace{-\frac{I_0}{2}\frac{\omega_1}{\left(\underline{s}+\alpha\right)^2+\omega_1^2}-\frac{U_0}{\omega_1 L}\cdot\frac{\omega_1}{\left(\underline{s}+\alpha\right)^2+\omega_1^2}}_{=0};\\ \alpha=\frac{R}{L},\ \omega_1=2\alpha,\ -\frac{I_0}{2}=\frac{U_0}{\omega_1 L}=\frac{RI}{\frac{R}{L}2L}=\frac{I}{2}.\end{cases}$$

$$\xrightarrow{\mathscr{L}^{-1}}i(t)=-I\cdot e^{-\alpha\left(t-t_0\right)}\cdot\cos\left[\omega_1\left(t-t_0\right)\right]\cdot h\left(t-t_0\right)$$



Slika 3b