2. Rezonancije:

Zadatak 1

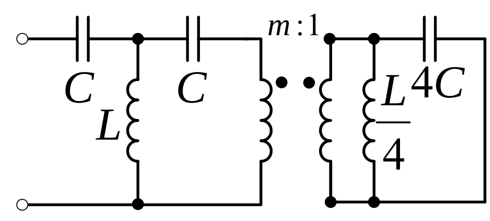
Mreža na slici 1. ima poznate parametre L, C, m=2.

a) odrediti ulaznu impedansu mreže $\underline{Z}(\underline{s}) = \underline{Z}(j\omega)$;

b) naći polove ulazne funkcije mreže;

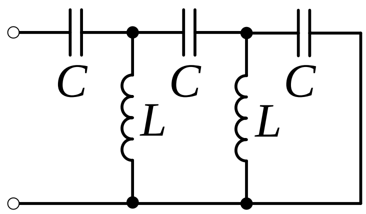
c) ako je mreža u ustaljenom prostoperiodičnom režimu, odrediti učestanosti rezonancije i antirezonancije;

d) nacrtati grafik reaktanse $X(\omega)$, ako važi uslov iz **c)**



Slika 1

a) radi lakšeg rešavanja mrežu možemo preslikati u ekvivalentnu mrežu:



$$\underline{Z}(\underline{s}) = \underline{Z}_C + \cfrac{1}{\underline{Y}_L + \cfrac{1}{\underline{Z}_C + \cfrac{1}{\underline{Y}_L + \cfrac{1}{\underline{Z}_C}}}},\ \underline{Z}_C = \frac{1}{\underline{s}C},\ \underline{Y}_L = \frac{1}{\underline{s}L},$$

$$\underline{Z}(\underline{s}) = \frac{1 + 4LC\underline{s}^2 + 3L^2C^2\underline{s}^4}{\underline{s}C(1 + 3LC\underline{s}^2 + L^2C^2\underline{s}^4)};$$

b)

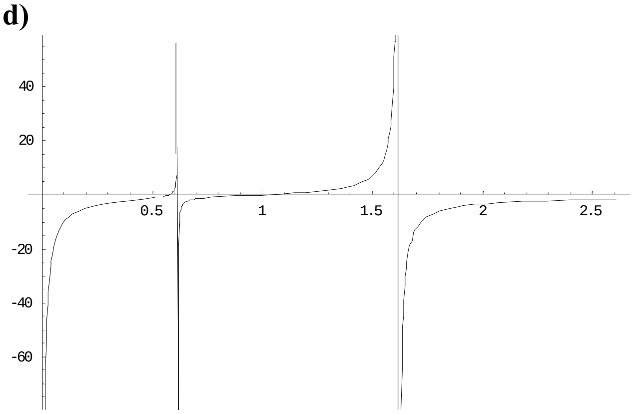
$$\left.\begin{array}{c} \underline{s}_{p1} = 0,\ \omega_0 = \dfrac{1}{\sqrt{LC}} \\ \underline{s}_{p2} = -j\omega_0\sqrt{\dfrac{3-\sqrt{5}}{2}},\ \underline{s}_{p3} = j\omega_0\sqrt{\dfrac{3-\sqrt{5}}{2}} \\ \underline{s}_{p4} = -j\omega_0\sqrt{\dfrac{3+\sqrt{5}}{2}},\ \underline{s}_{p5} = j\omega_0\sqrt{\dfrac{3+\sqrt{5}}{2}} \end{array}\right\} \text{polovi}$$

c)

$$\text{antirezonancije} = \left\{\begin{array}{c} \omega_{a1} = 0, \\ \omega_{a2} = \omega_0\sqrt{\dfrac{3-\sqrt{5}}{2}}, \\ \omega_{a3} = \omega_0\sqrt{\dfrac{3+\sqrt{5}}{2}} \end{array}\right\};$$

$$\text{rezonancije} = \left\{\begin{array}{c} \omega_{r1} = \dfrac{\omega_0}{\sqrt{3}}, \\ \omega_{r2} = \omega_0, \\ \omega_{r2} = \infty \end{array}\right\}.$$

d)



Izgled funkcije normirane na ω_0 po x osi,

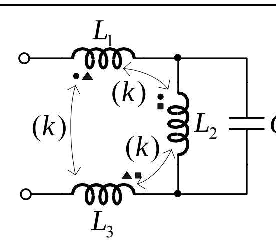
Zadatak 2

Mreža na slici 2 sadrži linearan transformator poznatih parametara $L_1 = L_2 = L_3 = L$, $k = ½$, i kondenzator kapacitivnosti C. Odrediti:

a) Kompleksnu ulaznu impedansu mreže.

b) Rezonantne i antirezonantne učestanosti mreže

Grafik ulazne reaktanse mreže u funkciji kružne učestanosti.



Slika 2

Rešenje:

$a)\ \ \underline{Z}_{ul} = L\underline{s}\dfrac{6 + 2LC\underline{s}^2}{1 + LC\underline{s}^2} = jX(\omega);$

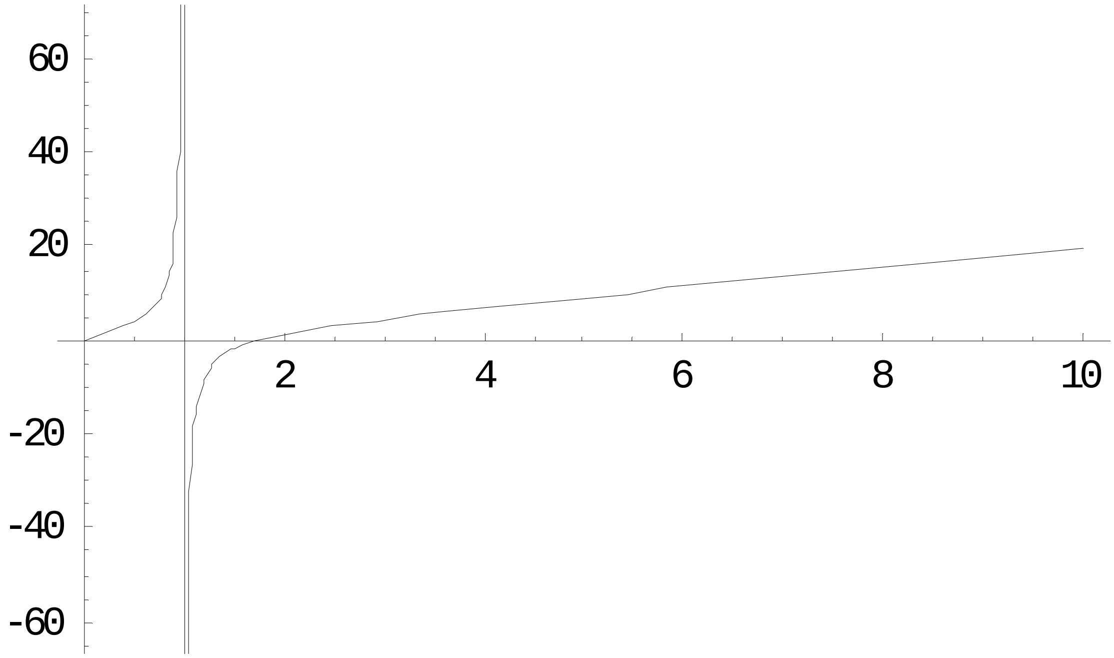
$b)\ \ \underline{s}_{r0} = 0,\ \ \underline{s}_{r1,2} = \pm j\dfrac{\sqrt{3}}{\sqrt{LC}} = \pm j\sqrt{3}\omega_0;$

$c)\ \ X(\omega) = L\omega\dfrac{6 - 2LC\omega^2}{1 - LC\omega^2};$

$\omega_{r0} = 0,\ \ \omega_{r1,2} = \pm\sqrt{3}\omega_0$

$\omega_{a1,2} = \omega_0,\ \ \omega_{a+\infty} = +\infty,\ \ \omega_{a-\infty} = -\infty$

$\underline{s}_{a1,2} = \pm j\dfrac{1}{\sqrt{LC}} = \pm j\omega_0;$



Izgled funkcije normirane na ω_0 po x osi.

Zadatak 3

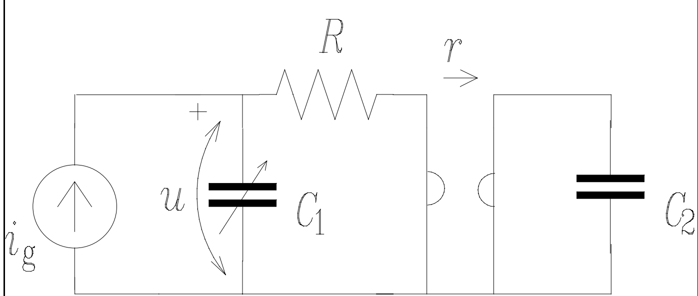
U kolu poznatih parametara R, $r = R$ i C_2, slika 3, deluje prostoperiodičan generator struje $i_g = \sqrt{2}I\cos\omega t$. Režim je ustaljen. Odrediti:

a) Kompleksan napon na krajevima generatora.

b) Vrednost kapacitivnosti C_1 pri kojoj u kolu nastaje fazna antirezonancija.

c) Efektivnu vrednost napona $u(t)$ pri ispunjenom uslovu pod b).

Numeričke vrednosti (zameniti u krajnjem rezultatu): $R = 10\,\text{k}\Omega$, $C_2 = 10\,\text{nF}$, $I = 1\,\text{mA}$, $\omega = 10^4\, rad/s$.



Slika 3

Rešenje:

a) $\underline{U} = \underline{Z}_{ul}\underline{I}_g;$

$\underline{Y}_{ul} = G_{ul} + jB_{ul};$

$$G_{ul} = \frac{1}{R + R^3C^2\omega^2}, \quad B_{ul} = C_1 - \frac{C_2}{1 + R^2C_2^2\omega^2}$$

b) Uslov fazne rezonancije je: $B_{ul} = 0, \quad \Rightarrow C_1 = \dfrac{C_2}{1 + R^2C_2^2\omega^2} = 5nF;$

c) $U = 2RI = 20V.$

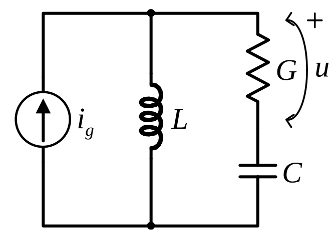
3. Ustaljen složenoperiodičan režim:

Zadatak 1

U kolu na slici 1 poznati su parametri G i ω, kao i struja generatora: $i_g = I^{(1)}\cos\omega t + I^{(3)}\cos 3\omega t$.

a) Odrediti parametre L i C tako da osnovni harmonik napona otpornika ne zavisi od G, i da amplituda trećeg harmonika struje u otporniku iznosi 90% trećeg harmonika struje izvora .

b) Izračunati trenutnu vrednost napona otpornika G u ustaljenom režimu, kao i snagu deformacije kondenzatora, ako su ispunjeni uslovi pod a).



Slika 1

Rešenje:

a) za k-ti harmonik se dobije $\underline{U}^{(k)} = \dfrac{j\omega kL\underline{I}_g^{(k)}}{1+jG(k\omega L - \dfrac{1}{k\omega C})}$, iz uslova zadatka da prvi harmonik ne zavisi od G dobije se da je $\omega L - \dfrac{1}{\omega C} = 0$, dok za treći harmonik važi uslov

$$\frac{3\omega LGI_g^{(3)}}{\sqrt{1+G^2(3\omega L - \dfrac{1}{3\omega C})^2}} = 0,9I^{(3)},\ \text{i dobijemo da je}\ C = \frac{2G}{\omega},\ \ L = \frac{1}{2G\omega}.$$

b) Iz prethodnog je: $\underline{U}^{(k)} = \dfrac{\dfrac{jk}{2G}\underline{I}_g^{(k)}}{1+j\dfrac{1}{2}(k-\dfrac{1}{k})}$, odakle je: $\underline{U}^{(1)} = \dfrac{jI_g^{(1)}}{2G} \Rightarrow u^{(1)}(t) = \dfrac{I^{(1)}}{2G}\cos(\omega t + \dfrac{\pi}{2})$,

$$\underline{U}^{(3)} = \frac{j3I_g^{(3)}}{2G(1+j\dfrac{4}{3})} = 0,9\frac{I_g^{(3)}}{G}e^{j\mathrm{arctg}\frac{3}{4}} \Rightarrow u^{(3)} = 0,9\frac{I^{(3)}}{G}\cos(3\omega t + \mathrm{arctg}\frac{3}{4}),$$

$$u(t) = u^{(1)}(t) + u^{(3)}(t).$$

Snaga deformacije kondenzatora je:

$$D_C = \frac{1}{2}\left|U_C^{(1)}I_C^{(3)} - U_C^{(3)}I_C^{(1)}\right| = \frac{1}{2}\left|\frac{1}{\omega C}I_C^{(1)}I_C^{(3)} - \frac{1}{3\omega C}I_C^{(3)}I_C^{(1)}\right| = \frac{1}{3\omega C}I_C^{(1)}I_C^{(3)},$$

$$I_C^{(1)} = I_G^{(1)} = \frac{I_g^{(1)}}{2},\ \ I_C^{(3)} = I_G^{(3)} = 0,9I_g^{(3)},$$

$$D_C = \frac{3I_g^{(1)}I_g^{(3)}}{40G}.$$

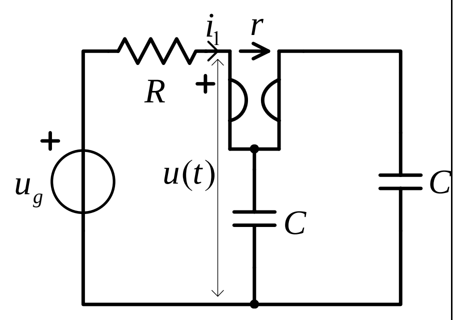
Zadatak 3

Parametri r, C i R=4r/3, kola sa slike 3, su poznati. Odrediti:

a) Ulaznu impedansu mreže vezane za krajeve generatora , $\underline{Z}_{ul}(\underline{s})=U_g/I_1$.

b) Trenutnu vrednost napona $u(t)$ ako je $u_g(t)=U_g^{(1)}\cos\omega_1 t+U_g^{(3)}\cos 3\omega_1 t$, sa $\omega_1=1/(3RC)$.

c) Sve snage generatora.



Slika 3.

Rešiti 3 zadatak sami!

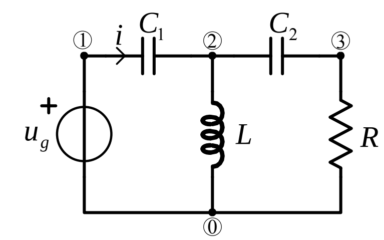
Zadatak 4

Parametri kola sa slike 4 su poznati, $C_1=C_2=C$, $R=\sqrt{\dfrac{L}{C}}$,

$u_g(t)=U+\sqrt{2}U\cos(\dfrac{1}{\sqrt{LC}}t)$. Režim rada je ustaljen.

a) Odrediti trenutnu vrednost struje generatora $i(t)$.

b) Kolika je srednja snaga otpornika?



Slika 4

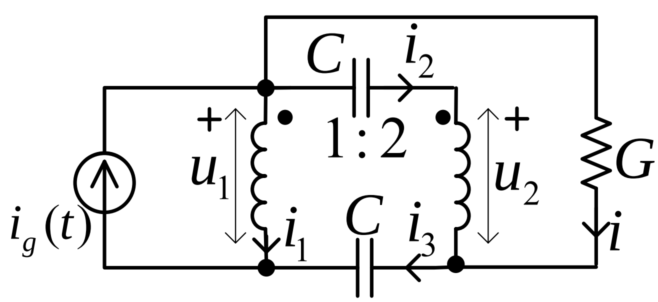
Rešenje:

a) $\underline{Z}_{ekv}^{(1)}=R,\quad \underline{Z}_{ekv}^{(0)}=\infty,\quad \Rightarrow i(t)=\sqrt{2}\dfrac{U}{R}\cos(\omega t);\quad R=\sqrt{\dfrac{C}{L}},\quad \omega=\dfrac{1}{\sqrt{LC}};$

b) $P_g=\dfrac{U^2}{R}$, Pošto su L i C elementi bez gubitaka.

Zadatak 5

Ako je u kolu sa slike 5 uspostavljen ustaljen složeno-periodični režim, gde je struja strujnog generatora oblika $i_g(t)=I_0+I_1\cos\omega t$, odrediti trenutnu vrednost struje $i(t)$ u otporniku provodnosti G.



Slika 5.

Rešenje: Analizom kola dobijemo sistem jednačina:

$$\underline{U}_2=2\underline{U}_1,\quad \underline{I}_1=-2\underline{I}_2, \qquad \text{KE}$$

$$\underline{I}_1+\underline{I}_2+\underline{I}=\underline{I}_g,\quad \underline{I}_3-\underline{I}_2-\underline{I}=0, \qquad \text{KZS}$$

$$\underline{U}_1=\frac{i}{G}+\frac{1}{\underline{s}C}(\underline{I}+\underline{I}_2)=\frac{\underline{I}_2}{\underline{s}C}+\underline{U}_2+\frac{\underline{I}_3}{\underline{s}C} \qquad \text{KZN}+\text{KE}$$

$$\frac{\underline{s}C\underline{I}}{G}+5\underline{I}=3\underline{I}_g,\quad \underline{s}=j\omega.$$

U ustaljenom složenoperiodičnom režimu struja k-tog harmonika je: $\underline{I}^{(k)}=\dfrac{3\underline{I}_g^{(k)}}{5+j\dfrac{Ck\omega}{G}},\quad (k=0,1)$.

gde je $\underline{I}_g^{(0)}=I_0,\quad \underline{I}_g^{(1)}=\dfrac{I_1}{\sqrt{2}}\Rightarrow \underline{I}^{(0)}=\dfrac{3}{5}I_0,\quad \underline{I}^{(1)}=\dfrac{3\dfrac{I_1}{\sqrt{2}}}{\sqrt{25+\dfrac{C^2\omega^2}{G^2}}}e^{-j\operatorname{arctg}\frac{C\omega}{5G}}$; pa je $i(t)=\dfrac{3}{5}I_0+\dfrac{3I_1}{\sqrt{25+\dfrac{C^2\omega^2}{G^2}}}\cos(\omega t-\operatorname{arctg}\dfrac{C\omega}{5G})$.

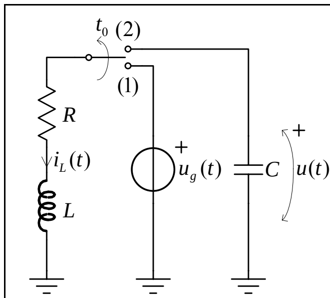
4. Laplasova transformacija:

Zadatak 1

Parametri kola sa slike 1 su poznati i važi da je $L = R^2C$, $\omega = R/L$, $u_g(t) = U_m\sin(\omega t)$. Prekidač je u položaju (1) i kolo je u ustaljenom režimu rada. U trenutku t_0, kada je struja kalema maksimalna, prekidač se prebacuje u položaj (2). Analizom u kompleksnom domenu Laplasove transformacije odrediti:

a) napon na kondenzatoru $u(t)$ za $t \ge 0$,

b) rad koji se ulaže u otpornik u intervalu (t_0^+, ∞).



slika 1

Rešenje:

a) $\underline{U}_g(\underline{s}) = \frac{U_m}{\sqrt{2}} e^{-j\frac{\pi}{2}}, \underline{I}_L = \frac{\underline{U}_g}{R + j\omega L} = \frac{\underline{U}_g}{R}\frac{1}{1+j} = \frac{U_m}{2R} e^{-j\frac{3\pi}{4}}, i_L(t) = \frac{U_m}{\sqrt{2}R}\cos\left(\omega t - \frac{3\pi}{4}\right), i_L(t_0^-) = \frac{U_m}{\sqrt{2}R}, t_0^- = \frac{3\pi}{4\omega}.$

$\tau = t - t_0,\ \underline{U}(\underline{s}) = R\underline{I}_L + \underline{U}_L,\ \underline{U}_L = \underline{s}L\underline{I}_L - LI_{L0},\ \underline{I}_L = -\underline{s}C\underline{U} + \underbrace{C\underline{U}_0}_{=0}.$

$$\underline{U}(\underline{s}) = \frac{-LI_{L0}}{1 + RC\underline{s} + (RC)^2\underline{s}^2} = \frac{-I_{L0}}{C}\frac{1}{\underline{s}^2 + \frac{\underline{s}}{RC} + \frac{1}{(RC)^2}} = \frac{-I_{L0}}{C\omega_1}\frac{\omega_1}{(\underline{s}+\alpha)^2 + \omega_1^2},$$

$$\xrightarrow{\mathscr{L}^{-1}} u(t) = \frac{-I_{L0}}{C\omega_1} e^{-\alpha(t-t_0)} \cdot \sin\left[\omega_1(t-t_0)\right], t \ge t_0,\ \alpha = \frac{1}{2\mathrm{RC}}, \omega_1 = \frac{\sqrt{3}}{2RC}.$$

b) Ukupna energija kola se tokom vremena troši na otporniku, a ona je jednaka energiji kalema u početnom trenutku, pa važi:

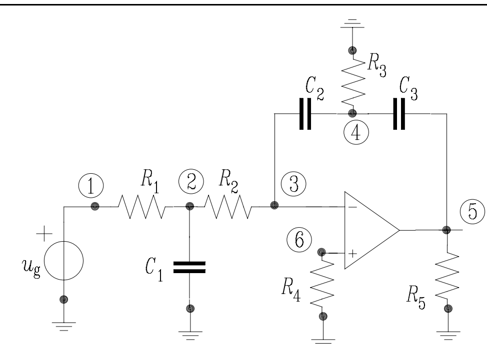
$$a(t_0, t) = \frac{1}{2}LI^2 = \frac{1}{2}R^2C\frac{U_m^2}{2R^2} = \frac{1}{4}CU_m^2$$

Zadatak 2

U kolu poznatih parametara $R_1 = R_2 = 2R$, $C_1 = 2C$, $R_3 = R_4 = R_5 = R$, $C_2 = C_3 = C$, slika 2, odrediti:

a) funkciju mreže $\underline{M}(\underline{s}) = \underline{V}_5(\underline{s})/\underline{U}_g(\underline{s})$.

b) impulsni odziv (Grinovu funkciju) potencijala čvora 5.



slika 2

Rešenje: Rešićemo kolo pomoću MNA

$2:\ \frac{\underline{V}_2 - \underline{V}_1}{2R} + 2\underline{s}C\underline{V}_2 + \frac{\underline{V}_2 - \underline{V}_3}{2R} = 0, \qquad \underline{V}_1 = \underline{U}_g;$

$3:\ \frac{\underline{V}_3 - \underline{V}_2}{2R} + \underline{s}C(\underline{V}_3 - \underline{V}_4) = 0, \qquad \underline{V}_3 = \underline{V}_6 = 0;$

$4:\ \underline{s}C(\underline{V}_4 - \underline{V}_3) + \frac{\underline{V}_4}{R} + \underline{s}C(\underline{V}_4 - \underline{V}_5) = 0.$

a) $\underline{M}(\underline{s}) = -\frac{1}{4R^2C^2\underline{s}^2}$

b) $g(t) = -\frac{t}{4R^2C^2}\mathrm{h}(t)$

Zadatak 3

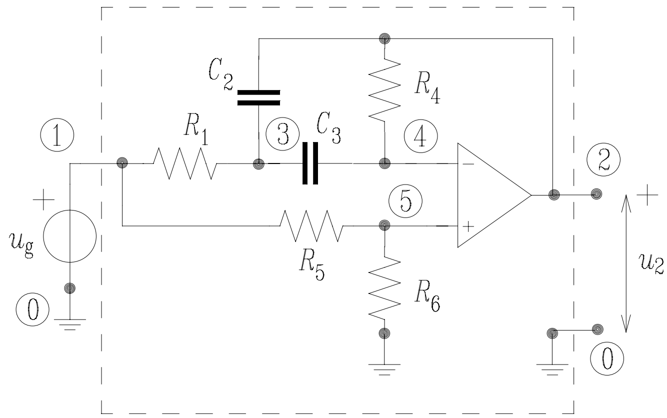
Parametri kola sa slike 3 su poznati. Kapacitivnosti kondezatora su C, $R_1 = R_4 = R_6 = R$, $R_5 = 2R$. Odrediti:

a) funkciju mreže $\underline{T}(\underline{s}) = \underline{U}_2(\underline{s})/\underline{U}_g(\underline{s})$,

b) njene nule i polove,

c) graničnu vrednost napona $u_2(t)$, kada $t \to \infty$, ako je impulsna ekscitacija $u_g(t) = \Phi\delta(t)$, i

d) Efektivnu vrednost napona $u_2(t)$ u ustaljenom režimu, ako je eksitacija $u_g(t) = U + U\sin(\frac{t}{3RC}) + U\cos(\frac{t}{CR})$.



Slika 3

Rešenje:

a) $\underline{T}(\underline{s}) = \dfrac{(\underline{s}RC)^2 + 1}{3[(\underline{s}RC)^2 + 2\underline{s}RC + 1]}$;

b) $\underline{s}_{1,2nule} = \pm j\omega_0$; $\underline{s}_{1,2polovi} = -\omega_0$; $\omega_0 = \dfrac{1}{RC}$.

c) $\lim\limits_{t\to\infty} u_2(t) = \lim\limits_{\underline{s}\to 0} \underline{s}\underline{U}_2(\underline{s}) = 0$.

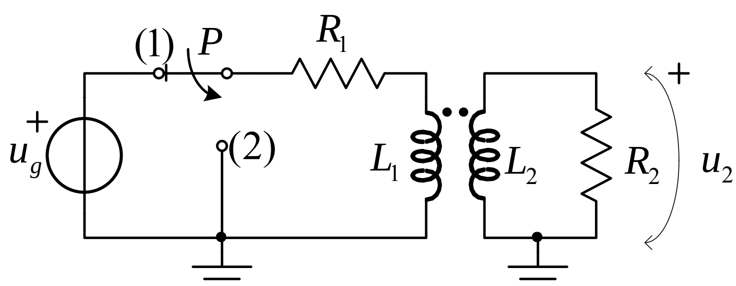
d) $U_2 = \sqrt{U_2^{(0)^2} + U_2^{(1)^2} + U_2^{(3)^2}} = \dfrac{U}{5}\sqrt{\dfrac{11}{3}}$.

Zadatak 4 Parametri kola sa slike 4 su poznati: $u_g(t) = E, L_1 = 4L, L_2 = L, R_1 = 4R, R_2 = R.$ Režim rada kola je ustaljen i $k = \dfrac{1}{2}$. U trenutku t_0 prekidač P se prebacuje u položaj (2). Odrediti:

a) Prirodne početne uslove u trenutku t_0^-,

b) Trenutnu vrednost napona otpornika R_2, za $t \geq t_0$, analizom u kompleksnom domenu Laplasove transformacije,

c) Kolika je granična vrednost ovog napona posle beskonačno dugog vremena, ispitati da li je komutacija regularna.

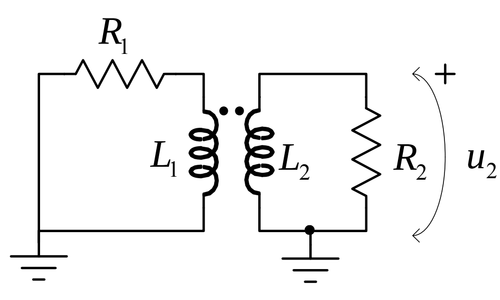


Slika 4

Rešenje:

a) Pošto se kalemovi ponašajukao kratak spoj u jednosmernom režimu dobijemo. $i_1(t_0^-) = \dfrac{E}{4R}, i_2(t_0^-) = 0.$

b) Posle prebacivanja prekidača u položaj 2 sedobije sledeće kolo:



Ako uvedemo da je $\tau = t - t_0, i_1(0^-) = I_{10} = \dfrac{E}{4R}, i_2(0^-) = I_{20} = 0.$ Sada postavimo sistem jednačina:

$u_1 = L_1Di_1 + L_{12}Di_2,\ \ u_1 = -R_1i_1; \xrightarrow{\mathscr{L}} \underline{U}_1 = \underline{s}L_1\underline{I}_1 - L_1I_{10} + \underline{s}L_{12}\underline{I}_2 - L_{12}\cancel{I_{20}},\ \underline{U}_1 = -R_1\underline{I}_1;$

$u_2 = L_{12}Di_1 + L_2Di_2,\ u_2 = -R_2i_2; \xrightarrow{\mathscr{L}} \underline{U}_2 = \underline{s}L_{12}\underline{I}_1 - L_{12}I_{10} + \underline{s}L_2\underline{I}_2 - L_2\cancel{I_{20}},\ \underline{U}_2 = -R_2\underline{I}_2;$

$$\underline{U}_2 = \frac{-4\dfrac{R^2}{L}I_{10}}{3\underline{s}^2 + 8\dfrac{R}{L}\underline{s} + 4\left(\dfrac{R}{L}\right)^2} = \frac{-4RaI_{10}}{3\underline{s}^2 + 8a\underline{s} + 4a^2},\ a = \frac{R}{L};\ \ 3\underline{s}^2 + 8a\underline{s} + 4a^2 = 0 \Rightarrow \underline{s}_1 = -\frac{2}{3}a, \underline{s}_1 = -2a;$$

$$\underline{U}_2 = \frac{k_1}{(\underline{s} - \underline{s}_1)} + \frac{k_2}{(\underline{s} - \underline{s}_2)} = \frac{k_1}{\left(\underline{s} + \dfrac{2}{3}a\right)} + \frac{k_2}{(\underline{s} + 2a)};$$

$$k_1 = \lim_{\underline{s}\to\underline{s}_1}\left[\underline{U}_2(\underline{s})(\underline{s} - \underline{s}_1)\right] = \lim_{\underline{s}\to\underline{s}_1}\left[\left(\frac{-4RaI_{10}}{3\underline{s}^2 + 8a\underline{s} + 4a^2}\right)\left(\underline{s} + \frac{2}{3}a\right)\right] = \lim_{\underline{s}\to\underline{s}_1}\frac{-4RaI_{10}}{3(\underline{s} + 2a)} = -RI_{10};$$

$$k_2 = \lim_{\underline{s}\to\underline{s}_2}\left[\underline{U}_2(\underline{s})(\underline{s} - \underline{s}_2)\right] = \lim_{\underline{s}\to\underline{s}_1}\left[\left(\frac{-4RaI_{10}}{3\underline{s}^2 + 8a\underline{s} + 4a^2}\right)(\underline{s} + 2a)\right] = \lim_{\underline{s}\to\underline{s}_1}\frac{-4RaI_{10}}{3\left(\underline{s} + \dfrac{2}{3}a\right)} = RI_{10};$$

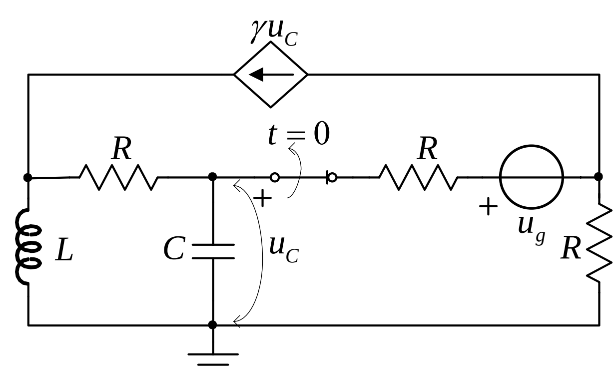
$$u_2(t) = \left[-RI_{10}e^{-\frac{2}{3}a(t-t_0)} + RI_{10}e^{-2a(t-t_0)}\right]h(t).$$

Zadatak 5 U kolu poznatih parametara $L, C, R > 0, \gamma = \dfrac{1}{R},$ deluje konstantan generator napona $u_g(t) = U.$ Prekidač P je zatvoren i u kolu je ustaljen režim. U trenutku $t = 0$ prekidač se otvara.

a) Odrediti prirodne početne uslove u trenutku $t = 0^-$,

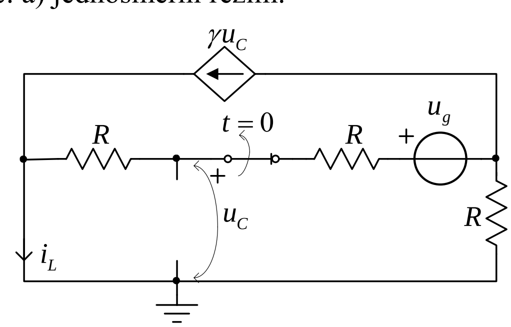
b) Odrediti kompleksan napon $\underline{U}_C(\underline{s})$ u domenu Laplasove transformacije,

c) Primenom inverzne Laplasove transformacije odrediti $u_C(t),$ ako je $R = \sqrt{\dfrac{L}{C}}.$



Slika 5

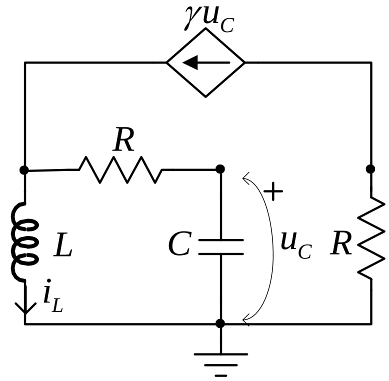
Rešenje: a) jednosmerni režim:



$$u_C(0^-) = \frac{U}{4} = U_0;$$

$$i_C(0^-) = \frac{U}{2R} = I_0;$$

b) posle otvaranja prekidača



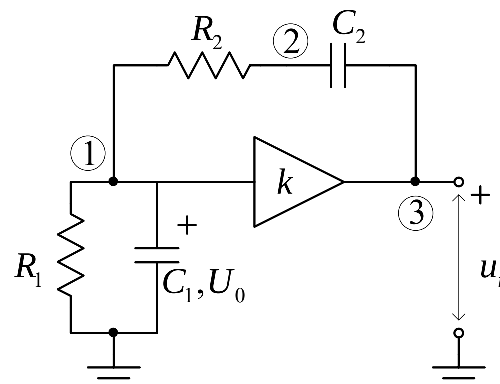
$$\left.\begin{cases} R\underline{I}_C + \underline{U}_C - \underline{U}_L = 0 \\ \gamma \underline{U}_C = \underline{I}_L + \underline{I}_C \\ \underline{U}_C = \underline{s}L\underline{I}_L - LI_0 \\ \underline{I}_C = \underline{s}C\underline{U}_C - CU_0 \end{cases}\right\} \Rightarrow \underline{U}_C(\underline{s}) = \frac{RCU_0 + LCU_0\underline{s} - LI_0}{\underline{s}^2 + \frac{RC-\gamma L}{LC}\underline{s} + \frac{1}{LC}};$$

$$\gamma = \frac{1}{R}, R = \sqrt{\frac{L}{C}}, \omega = \frac{1}{\sqrt{LC}} = \frac{1}{RC},$$

$$\underline{U}_C = -\frac{U}{4}\cdot\frac{\omega}{\underline{s}^2+\omega^2} + \frac{U}{4}\cdot\frac{\omega}{\underline{s}^2+\omega^2} \Rightarrow u_C(t) = \frac{U}{4}\left[\cos(\omega t) - \sin(\omega t)\right], t \geq 0.$$

Zadatak 5 Dato je kolo sa idealnim naponskim pojačavačem pojačanja *k*. Odrediti:

a) Kompleksni napon izlaza $\underline{U}_i(\underline{s})$ u domenu Laplasove transformacije,
b) Sopstvene učestanosti napona $\underline{U}_i(\underline{s})$, $\underline{s}_m$ i odziv $u_i(t)$, ako su parameri $R_1 = R_2 = R,\ C_1 = C_2 = C$, i ako pojačanje k redom uzima vrednosti $k = 0.5, 1, 2, 3, 4, 5$.



Slika 5

$\underline{s} = \sigma + j\omega,$

$$1:\ \frac{\underline{V}_1}{R_1} + \underline{s}C_1\underline{V}_1 - C_1U_0 + \frac{\underline{V}_1 - \underline{V}_2}{R_2} = 0,\ \ \underline{V}_3 = k\underline{V}_1;$$

$$2:\ \frac{\underline{V}_2 - \underline{V}_1}{R_2} + \underline{s}C_2\left(\underline{V}_2 - \underline{V}_3\right) = 0,\qquad \underline{V}_3 = \underline{U}_i;$$

Za $R_1 = R_2 = R,\ C_1 = C_2 = C$ dobijemo:

$$\underline{U}_i = \frac{kU_0\left(\underline{s}+\omega_0\right)}{\underline{s}^2 + 2\alpha\underline{s} + \omega_0^2}, \alpha = \frac{(3-k)}{2}\omega_0;$$

$$\underline{s}_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

$$\underline{U}_i = \frac{kU_0\left(\underline{s} + \frac{1}{C_2R_2}\right)}{\underline{s}^2 + \left(\frac{1}{C_2R_2} + \frac{1}{C_1R_1} + \frac{1-k}{C_1R_2}\right) + \frac{1}{C_1R_1C_2R_2}}.$$

Za $k = 0.5$:

$$\alpha = \frac{2.5}{2}\omega_0;\ \ \underline{s}_1 = -0.5\omega_0,\ \underline{s}_2 = -2\omega_0;$$

$$\underline{U}_i = \frac{k_1}{\left(\underline{s} - \underline{s}_1\right)} + \frac{k_2}{\left(\underline{s} - \underline{s}_2\right)};$$

$$\left.\begin{cases} k_1 = \lim\limits_{\underline{s}\to\underline{s}_1}\left[\underline{U}_i(\underline{s})\left(\underline{s} - \underline{s}_1\right)\right] = \frac{U_0}{6}; \\ k_2 = \lim\limits_{\underline{s}\to\underline{s}_2}\left[\underline{U}_i(\underline{s})\left(\underline{s} - \underline{s}_2\right)\right] = \frac{U_0}{3}; \end{cases}\right\} \xrightarrow{\mathscr{L}^{-1}} \underline{U}_i(\underline{s}) = \frac{U_0}{6}\left(e^{-0.5\omega_0 t} + 2e^{-2\omega_0 t}\right)h(t).$$

Za $k=1$:

$$\alpha=\omega_0;\quad \underline{s}_{1,2}=-\omega_0;$$

$$\underline{U}_i=\frac{k_1}{\left(\underline{s}-\underline{s}_1\right)}+\frac{k_2}{\left(\underline{s}-\underline{s}_1\right)^2};$$

$$\left.\begin{cases} k_1=\lim\limits_{\underline{s}\to\underline{s}_1}\left[\underline{U}_i(\underline{s})\dfrac{d}{d\underline{s}}\left(\underline{s}-\underline{s}_1\right)^2\right]=U_0; \\ k_2=\lim\limits_{\underline{s}\to\underline{s}_2}\left[\underline{U}_i(\underline{s})\left(\underline{s}-\underline{s}_2\right)^2\right]=0; \end{cases}\right\}\xrightarrow{\mathcal{L}^{-1}}\underline{U}_i(\underline{s})=U_0e^{-\omega_0 t}h(t).$$

Za $k=2$:

$$\alpha=0.5\omega_0;\quad \underline{s}_{1,2}=-0.5\omega_0\pm j\frac{\sqrt{3}}{2}\omega_0;$$

$$\underline{U}_i=\frac{kU_0\left(\underline{s}+\omega_0\right)}{\underline{s}^2+2\alpha\underline{s}+\omega_0^2}=\frac{kU_0\left(\underline{s}+\omega_0\right)}{\left(\underline{s}+\alpha\right)^2+\omega_1^2}=\frac{kU_0\left(\underline{s}+\alpha\right)}{\left(\underline{s}+\alpha\right)^2+\omega_1^2}-\frac{kU_0\left(\omega_0-\alpha\right)}{\left(\underline{s}+\alpha\right)^2+\omega_1^2},$$

$$\xrightarrow{\mathcal{L}^{-1}}\underline{U}_i(\underline{s})=2U_0e^{-\alpha t}\left[\cos\left(\omega_1 t\right)+\frac{1}{\sqrt{3}}\sin\left(\omega_1 t\right)\right]h(t).$$

Za $k=3$:

$$\alpha=0;\quad \underline{s}_{1,2}=\pm j\omega_0;$$

$$\underline{U}_i=\frac{kU_0\left(\underline{s}+\omega_0\right)}{\underline{s}^2+\omega_0^2}\xrightarrow{\mathcal{L}^{-1}}\underline{U}_i(\underline{s})=3U_0\left[\cos\left(\omega_0 t\right)+\sin\left(\omega_0 t\right)\right]h(t).$$

Za $k=4$:

$$\alpha=-0.5\omega_0;\quad \underline{s}_{1,2}=\omega_0\pm j\frac{\sqrt{3}}{2}\omega_0;$$

$$\underline{U}_i=\frac{kU_0\left(\underline{s}+\omega_0\right)}{\underline{s}^2+2\alpha\underline{s}+\omega_0^2}=\frac{kU_0\left(\underline{s}+\omega_0\right)}{\left(\underline{s}+\alpha\right)^2+\omega_1^2}=\frac{kU_0\left(\underline{s}+\alpha\right)}{\left(\underline{s}+\alpha\right)^2+\omega_1^2}-\frac{kU_0\left(\omega_0-\alpha\right)}{\left(\underline{s}+\alpha\right)^2+\omega_1^2},$$

$$\xrightarrow{\mathcal{L}^{-1}}\underline{U}_i(\underline{s})=4U_0e^{\omega_0 t}\left[\cos\left(\omega_1 t\right)+\sqrt{3}\sin\left(\omega_1 t\right)\right]h(t).$$

Za $k=5$:

$$\alpha=-\omega_0;\quad \underline{s}_{1,2}=\omega_0;$$

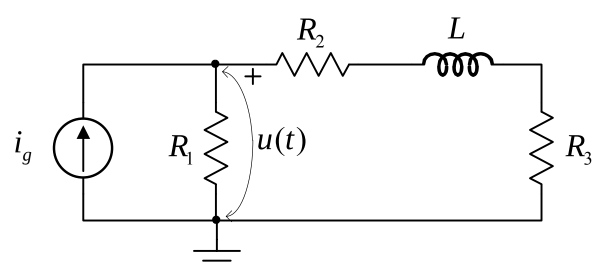
$$\underline{U}_i=\frac{k_1}{\left(\underline{s}-\underline{s}_1\right)}+\frac{k_2}{\left(\underline{s}-\underline{s}_1\right)^2};$$

$$\left.\begin{cases} k_1=\lim\limits_{\underline{s}\to\underline{s}_1}\left[\underline{U}_i(\underline{s})\dfrac{d}{d\underline{s}}\left(\underline{s}-\underline{s}_1\right)^2\right]=5U_0; \\ k_2=\lim\limits_{\underline{s}\to\underline{s}_2}\left[\underline{U}_i(\underline{s})\left(\underline{s}-\underline{s}_2\right)^2\right]=5U_0\omega_0; \end{cases}\right\}\xrightarrow{\mathcal{L}^{-1}}\underline{U}_i(\underline{s})=5U_0\left[e^{\omega_0 t}+2\omega_0 te^{\omega_0 t}\right]h(t).$$

Zadatak 6 Parametri kola na slici 6 su poznati i iznose $R_1=R_2=R_3=R>0,\ i_g(t)=Ie^{-at}h(t), a>0, I>0,$ i važi da je $L=\dfrac{4R}{a}$. Odrediti:

a) Kompleksan napon $\underline{U}(\underline{s})$ u domenu Laplasove transformacije,
b) Odziv napona $u(t)$,
c) Trenutak vremena kad napon $u(t)$ ima minimalnu vrednost i vrednost tog minimuma,

Ispitati da li se promenom induktivnosti kalema može ostvariti idealna antirezonancija



Slika 6

Rešenje: Prvo nađemo impedansu koju vidi generator, u kompleksnom domenu, jer nemamo početnu energiju:

$$\underline{U}(\underline{s}) = \underline{Z}(\underline{s})\underline{I}_g(\underline{s}) = \frac{R(2R+\underline{s}L)}{R+(2R+\underline{s}L)}\underline{I}_g(\underline{s}),\ \underline{I}_g(\underline{s}) = \frac{1}{\underline{s}+a}; \underline{U}(\underline{s}) = \frac{2RI(a+2\underline{s})}{(\underline{s}+a)(3a+4\underline{s})},$$

$$\underline{U}_i = \frac{k_1}{(\underline{s}-\underline{s}_1)} + \frac{k_2}{(\underline{s}-\underline{s}_2)};\ \underline{s}_1 = -a,\ \underline{s}_2 = -\frac{3a}{4};$$

$$\left.\begin{cases} k_1 = \lim\limits_{\underline{s}\to\underline{s}_1}\left[\underline{U}_i(\underline{s})(\underline{s}-\underline{s}_1)\right] = 2IR \\ k_2 = \lim\limits_{\underline{s}\to\underline{s}_2}\left[\underline{U}_i(\underline{s})(\underline{s}-\underline{s}_2)\right] = -IR \end{cases}\right\} \xrightarrow{\mathscr{L}^{-1}} \underline{U}_i(\underline{s}) = IR\left(e^{-\frac{3a}{4}t} - 2e^{-at}\right)h(t).$$

$$\frac{du(t)}{dt} = 0,\ t_m = \frac{4a}{a^2}\ln\frac{8}{3} = \frac{4}{a}\ln\frac{8}{3},\ u(t_m) = -IR\frac{27}{2048};$$

Ako je $\underline{s} = -\frac{3R}{L} = -a,$ može doći do idealne antirezonancije, jer je ravan $\sigma + j\omega$ pa je pobuda pseudoperiodična, iako je u antirezonanciji, amplituda napona ipak opada kao eksponencijalna funkcija.

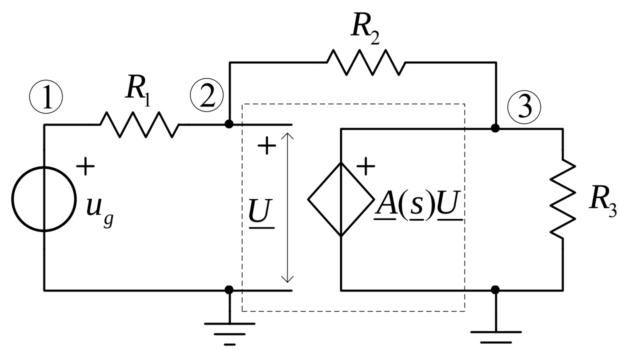
Zadatak 7 Kolo na slici 7 ima poznate parametre R_1, R_2, R_3. Pojačanje naponskog generatora kontrolisanog naponom je $\underline{A}(\underline{s}) = \frac{pa}{p+\underline{s}}$, gde su $a \neq 0, p > 0,$ i $p, a \in \mathrm{R},\ \underline{s} = \sigma + j\omega.$

a) Odrediti transmitansu napona $\underline{M}(\underline{s}) = \frac{\underline{V}_3(\underline{s})}{\underline{U}_g(\underline{s})}$,

b) Odrediti trenutnu vrednost napona $V_3(t)$ pomoću Laplasove transformacije,

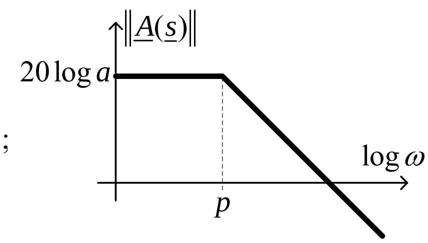
c) Naći vezu parametara tako da odziv bude konačan za $t \to \infty$,

d) Kolika je transmitansa $\underline{M}(\underline{s})$ kada $|a| \to \infty$.



Slika 7

Rešenje: Ako nacrtamo Bode-ov dijagram pojačanja $\underline{A}(\underline{s})$,



MNA:

$$\left.\begin{cases} 2:\ \dfrac{\underline{V}_2-\underline{V}_1}{R_1} + \dfrac{\underline{V}_2-\underline{V}_3}{R_2} = 0, \\ \underline{V}_1 = \underline{U}_g,\ \underline{V}_3 = A\underline{V}_2, \end{cases}\right\} \Rightarrow \underline{M}(\underline{s}) = \underbrace{ap\frac{R_2}{R_1+R_2}}_{=k}\frac{1}{\underline{s} + \underbrace{\frac{(1-a)R_1+R_2}{R_1+R_2}p}_{=\alpha}};$$

$$\underline{M}(\underline{s}) = \frac{k}{\underline{s}+\alpha}.$$

$$\underline{U}_g(\underline{s}) = \frac{U}{\underline{s}},\ \underline{V}_3(\underline{s}) = \underline{M}(\underline{s})\underline{U}_g(\underline{s}) = \frac{k}{\underline{s}+\alpha}\frac{U}{\underline{s}} = \frac{k_1}{\underline{s}+\alpha} + \frac{k_2}{\underline{s}}.$$

$$k_1 = -\frac{Uk}{\alpha},\ k_2 = \frac{Uk}{\alpha}$$

$$V_3(t) = \frac{Uk}{\alpha}\left(1-e^{-\alpha t}\right)h(t).$$

Da bi odziv bio konačan treba biti ispunjeno $a < 1 + \frac{R_2}{R_1}$.

$$\lim_{|a|\to\infty}\underline{M}(\underline{s}) = \frac{R_2}{R_1}.$$