Вероватноћа и статистика - домаћи задаци 2009.
одсеци: ОТ, ОЕ, ОГ, ОС, ОФ

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Решења

**1** Скуп исхода $\Omega = \{11, 011, 101, 100, 010, 00\}$

1 - погодак $P(1) = \frac{2}{3}$

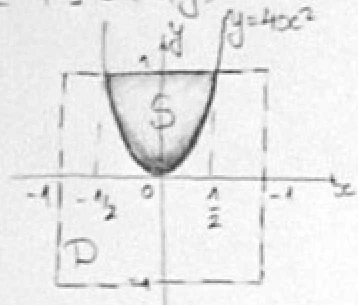
0 - промашај $P(0) = \frac{1}{3}$

A - догађај: циљ је погођен тачно једанпут: $\{100, 010\}$

$$P(A) = P(100) + P(010) = \frac{2}{3}\cdot\frac{1}{3}\cdot\frac{1}{3} + \frac{1}{3}\cdot\frac{2}{3}\cdot\frac{1}{3} = \frac{2\cdot 2}{27} = \frac{4}{27}$$

$$\boxed{P(A) = \frac{4}{27}}$$

**2** $[-1,1]^2 \ni (x,y)$




$$P(y > 4x^2) = \frac{S}{D} = \frac{\frac{2}{3}}{4} = \frac{1}{6}$$

за $(x,y) \in [-1,1]^2$

$$S = \left[\frac{1}{2} - \left(-\frac{1}{2}\right)\right] 1 - \int_{-1/2}^{1/2} 4x^2 dx =$$

$$= 1 - 8\int_0^{1/2} x^2 dx = 1 - 8\cdot\frac{x^3}{3}\Big|_0^{1/2} =$$

$$= 1 - \cancel{8}\cdot\frac{1}{\cancel{8}}\cdot\frac{1}{3} = \frac{2}{3}$$

$D = 2\cdot 2 = 4$

$$\boxed{P(y > 4x^2) = \frac{1}{6}}$$

**3** $P(A) = 0{,}59$
$P(B) = 0{,}29$
$P(AB) = 0{,}19$

$$P(A \cup B) = P(A) + P(B) - P(AB) =$$
$$= 0{,}59 + 0{,}29 - 0{,}19 = 0{,}69$$

1) $P(A \setminus B)$


$A \setminus B \cup B = A \cup B$

$$P(A \setminus B \cup B) = P(A \setminus B) + P(B) - P(\overbrace{\cancel{A \setminus B}) \cdot B}^{0})$$

$$P(A \cup B) = P(A \setminus B) + P(B)$$

$$0{,}69 = P(A \setminus B) + 0{,}29$$

$$\boxed{P(A \setminus B) = 0{,}4}$$

2) $P(A^c B^c) = P((A \cup B)^c) = 1 - P(A \cup B) = 1 - 0{,}69 = 0{,}31$

де Морганове формуле

$$\boxed{P(A^c B^c) = 0{,}31}$$

3) $P((AB)^c) = P(A^c \cup B^c) = P(A^c) + P(B^c) - P(A^c B^c) = 1 - P(A) + 1 - P(B) - 0{,}31 =$

$= 2 - 0{,}31 - 0{,}59 - 0{,}29 = 0{,}81$

$$\boxed{P((AB)^c) = 1 - P(AB) = 0{,}81}$$

4

A – студент је положио I део испита
$B^c$ – студент је пао II део испита
B – студент је положио II део испита

$P(AB)$?

$P(A)=0{,}55$
$P(B^c|A)=0{,}2$

$P(AB)=P(A)\cdot P(B)$ ⊛

$$P(B^c|A)=\frac{P(B^cA)}{P(A)}$$

$$P(B^cA)=P(B^c|A)\cdot P(A)=0{,}2\cdot 0{,}55=0{,}11$$

⊛ $P(B^c)\cdot P(A)=0{,}11$

$$P(B^c)=\frac{0{,}11}{0{,}55}=\frac{1}{5}$$

$$P(B)=1-\frac{1}{5}=\frac{4}{5}$$

$$P(AB)=0{,}55\cdot\frac{4}{5}=0{,}44$$

⊛ овде се полагање I и II дела сматра независним догађајима

5

$A=\{2,4,6\}_1$
$B=\{2,4,6\}_2$
$C=\{11,22,33,44,55,66\}$

$P(C)=\frac{1}{36}\cdot 6=\frac{1}{6}$

$P(A)=\frac{1}{6}\cdot 3=\frac{1}{2}$

$P(B)=\frac{1}{6}\cdot 3=\frac{1}{2}$

1) по паровима

A и B јесу

A и C

$$P(C|A)=\frac{P(AC)}{P(A)}=\frac{\frac{3}{36}}{\frac{1}{2}}=\frac{6}{36}=\frac{1}{6}=P(C)$$

јесу независне

B и C

$$P(C|B)=\frac{P(BC)}{P(B)}=\frac{\frac{3}{36}}{\frac{1}{2}}=\frac{6}{36}=\frac{1}{6}=P(C)$$

јесу независне

2) независни у целини

$$P(ABC)=\frac{3}{36}$$

$$P(A)P(B)P(C)=\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{6}\neq\frac{3}{36}=P(ABC)$$

нису независни у целини

$\boxed{\text{УСЛОВ НЕЗАВИСНОСТИ } P(AB)=P(A)\cdot P(B)}$

по паровима

$\boxed{\text{УСЛОВ НЕЗАВИСНОСТИ } P(A_{i_1}A_{i_2}\cdots A_{i_k})=P(A_{i_1})\cdots P(A_{i_k})}$

за било којих k догађаја од n

у целини

**6**

$A=\{2,4,6\}$
$B=\{4,5,6\}$
$C=\{1,2,3,4\}$

1) $P(ABC)=P(A)P(B)P(C)$?

$P(ABC)=P(\{4\})=\frac{1}{6}\neq\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{4}{6}$

$P(A)=\frac{1}{2}=P(B)\quad P(C)=\frac{4}{6}$

ДА

2) $P(AC)=P(\{2,4\})=\frac{2}{6}=\frac{1}{3}$

$P(A)\cdot P(C)=\frac{1}{2}\cdot\frac{2}{3}$

$P(BC)=\frac{1}{6}\neq$

$P(B)P(C)=\frac{1}{2}\cdot\frac{2}{3}=\frac{1}{3}$

B и C нису независни у пару

1) ДА 2) НЕ јер нису независни по паровима

**7**


НЕ

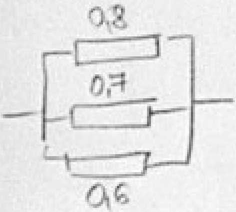
систем ради $\Leftrightarrow$ све компоненте раде

$p^6\geq 0{,}95$

$p\geq\sqrt[6]{0{,}95}=0{,}991488$

Поузданост мора бити бар 0,991488

**8**




систем не ради ако ни једна компонента не ради

Вероватноћа да систем не ради $P(A^c)$

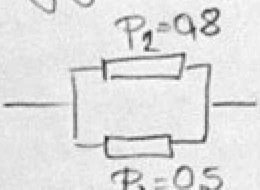
$P(A^c)=(1-0{,}8)(1-0{,}7)(1-0{,}6)=0{,}2\cdot 0{,}3\cdot 0{,}4$

$=\frac{3}{125}=0{,}024$

$P(A)=1-0{,}024=0{,}976$

Поузданост система је 97,6%

**9**




A - систем ради
B1 - ради прва компонента
B2 - ради друга компонента

$P(B1|A)$?

$P(B1|A)=\frac{P(AB_1)}{P(A)}$

$P(A)=1-(1-p_1)(1-p_2)=$
$=1-0{,}2\cdot 0{,}5=0{,}9$

$P(A|B_1)=\frac{P(AB_1)}{P(B_1)}\Rightarrow P(AB_1)=P(B_1)=p_1=0{,}5$

$P(A|B_1)=1$
ако B1 тада сигурно A

$P(B_1|A)=\frac{P(AB_1)}{P(A)}=\frac{0{,}5}{0{,}9}=\frac{5}{9}$

$\boxed{P(B_1|A)=0{,}556}$

**10**

$\boxed{\begin{matrix} 2Б \\ 3Ц \end{matrix}}$ I $\qquad \boxed{\begin{matrix} 4Б \\ 1Ц \end{matrix}}$ II

$$P(Ц) = P(I)\cdot P(Ц|I) + P(II)\,P(Ц|II) = 0{,}5\cdot\frac{\binom{3}{1}}{\binom{5}{1}} + 0{,}5\cdot\frac{\binom{1}{1}}{\binom{5}{1}} = 0{,}5\left(\frac{3}{5}+\frac{1}{5}\right) = 0{,}4$$

Вероватноћа је 0,4

**11**

$P(X=k) = Ck,\ k=1,2,3,4$

1) $P(X=1)+P(X=2)+P(X=3)+P(X=4)=1$

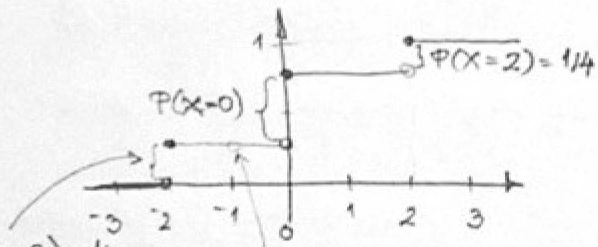
$C\cdot 1 + C\cdot 2 + C\cdot 3 + C\cdot 4 = 1$

$10C = 1 \quad \boxed{C=\frac{1}{10}}$

2) $P(-1{,}5<X<3{,}5) = P(X=1)+P(X=2)+P(X=3) = \frac{1+2+3}{10} = \frac{6}{10} = \boxed{\frac{3}{5}}$

**12**

$$F_X(x) = \begin{cases} 0, & x<-2 \\ 1/4, & -2\leq x<0 \\ 3/4, & 0\leq x<2 \\ 1, & x\geq 2 \end{cases}$$




$P(X=-2)=1/4$

ф-ја расподеле има скок

нема скока $\Rightarrow P(X=-1)=0$

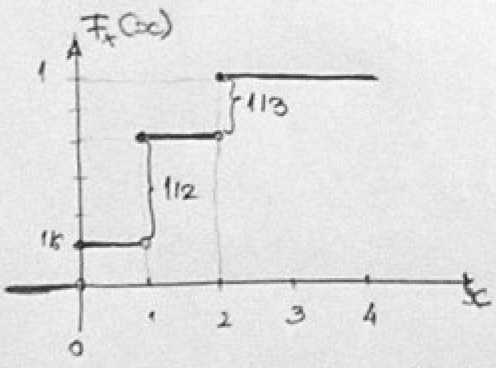
закон расподеле

$$X: \begin{pmatrix} -2 & 0 & 2 \\ 1/4 & 1/2 & 1/4 \end{pmatrix}$$

| |
|---|
| ОСОБИНЕ Ф-ЈЕ РАСПОДЕЛЕ |
| 1) $F(-\infty)=0 \quad F(+\infty)=1$ |
| 2) $F$ је неопадајућа |
| 3) леви лимес $F(x_-)=P(X<x)$ |
| 4) ф-ја је непрекидна здесна $F(x_+)=F(x)=P(X\leq x)$ |
| 5)* Величина скока је $F(x)-F(x_-)=P(X=x)$ |
| 6) $F$ је непрекидна у свакој тачки $x$ у којој је $P(X=x)=0$ |

**13**

$$X: \begin{pmatrix} 0 & 1 & 2 \\ 1/6 & 1/2 & 1/3 \end{pmatrix}$$




$\boxed{F_X(x) = P(X\leq x)}$

дефиниција

$$F_X(x) = \begin{cases} 0, & x<0 \\ 1/6, & 0\leq x<1 \\ 4/6, & 1\leq x<2 \\ 1, & x\geq 2 \end{cases}$$

$\boxed{14}$

$$f(x) = \begin{cases} cx^2, & x \in (0,1) \\ 0, & x \notin (0,1) \end{cases}$$

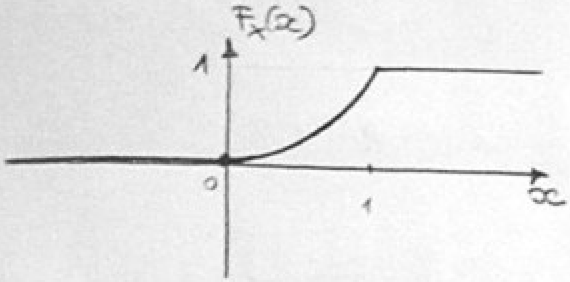
1) ВАЖИ ДА ЈЕ

$$\int_{-\infty}^{+\infty} f(x)\,dx = 1 \quad \int_{-\infty}^{+\infty} cx^2\,dx = 1 \quad c\int_0^1 x^2\,dx = 1$$

$$c\,\frac{x^3}{3}\Big|_0^1 = 1 \implies \frac{c}{3} = 1 \quad \boxed{c = 3}$$

2) $\boxed{F_X(x) = \int_{-\infty}^{x} f(t)\,dt}$

$$F_X(x) = \begin{cases} 0, & x \le 0 \\ \int_0^x 3x^2\,dx, & 0 < x < 1 \\ 1, & x \ge 1 \end{cases} = \begin{cases} 0, & x \le 0 \\ x^3, & 0 < x < 1 \\ 1, & x \ge 1 \end{cases}$$




$\boxed{15}$

$$f_X(x) = \begin{cases} C(x+1), & 2 < x < 4 \\ 0, & \text{иначе} \end{cases}$$

1) $C$? 2) $P(X < 3{,}2) = P(X \le 3{,}2) = F_X(3{,}2)$

1)
$$\int_{-\infty}^{+\infty} f_X(x)\,dx = 1 \quad c\int_2^4 (x+1)\,dx = 1 \quad c\,\frac{(x+1)^2}{2}\Big|_2^4 = 1$$

$$c \cdot \frac{25-9}{2} = 1 \quad \boxed{c = \frac{1}{8}}$$

2)
$$F_X(3{,}2) = \int_{-\infty}^{3{,}2} f_X(t)\,dt = \int_2^{3{,}2} \frac{1}{8}\cdot(t+1)\,dt = \frac{1}{8}\frac{(t+1)^2}{2}\Big|_2^{3{,}2} = \frac{4{,}2^2 - 9}{16} = \boxed{0{,}54}$$

$\boxed{16}$ $X \sim Exp(2)$ $\quad P(-3 < X \le 4) = F_X(4) - F_X(-3)$

$$f(x) = \begin{cases} 2e^{-2x}, & x \ge 0 \\ 0, & x < 0 \end{cases}$$

$$P(X \le -3) = F_X(-3) = \int_{-\infty}^{-3} f(x)\,dx = 0$$

$$P(X \le 4) = F_X(4) = \int_{-\infty}^{4} f(x)\,dx = \int_0^4 2e^{-2x}\,dx = -e^{-2x}\Big|_0^4 = -e^{-8} + 1 = 1 - e^{-8}$$

$$\boxed{P(-3 < X \le 4) = 1 - e^{-8}}$$

**17**

| X \ Y | -1 | 0 | 1 | |
|---|---|---|---|---|
| -1 | 1/8 | 0 | c | 3/8 |
| 1 | 1/2 | 1/4 | 0 | 6/8 |
| | 5/8 | 1/4 | 1/8 | |

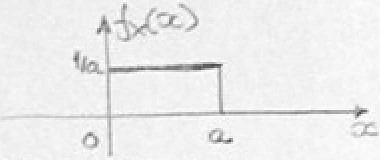
$\frac{1}{2}+\frac{1}{8}$

$1/8 + 0 + c + 1/2 + 1/4 + 0 = 1$

$\frac{7}{8} + c = 1 \Rightarrow \boxed{c = \frac{1}{8}}$

$X : \begin{pmatrix} -1 & 1 \\ 1/4 & 3/4 \end{pmatrix}$ $\quad Y : \begin{pmatrix} -1 & 0 & 1 \\ 5/8 & 2/8 & 1/8 \end{pmatrix}$

**18** $X \sim Unif(0,a)$ $\quad P(X<1,5) = 3/4$




$\boxed{\text{ПП} : a>0}$

$f_X(x) = \begin{cases} 0, & x<0 \\ \frac{1}{a}, & 0 \le x \le a \\ 0, & x>a \end{cases}$ $\quad F_X(x) = \begin{cases} 0, & x<0 \\ \frac{x}{a}, & 0\le x \le a \\ 1, & x>a \end{cases}$

$P(X<1,5) = F_X(1,5) = \int_{-\infty}^{1,5} f_X(x)\,dx = \frac{3}{4} \Rightarrow a > 1,5$

$\int_0^{1,5} \frac{1}{a}\,dx = \frac{3}{4}$ $\quad \frac{x}{a}\Big|_0^{1,5} = \frac{3}{4}$ $\quad \frac{3}{2a} = \frac{3}{4}$ $\quad \boxed{a=2}$

**19** $X : \begin{pmatrix} -2 & 2 & 5 \\ 1/4 & 1/2 & c \end{pmatrix}$ $\quad 1/4 + 1/2 + c = 1 \Rightarrow \boxed{c = \frac{1}{4}}$

$Y = X^2$ $\quad Y \in \{4, 25\}$ $\quad Y = \begin{pmatrix} 4 & 25 \\ 1/4 + 1/2 & 1/4 \end{pmatrix} = \begin{pmatrix} 4 & 25 \\ 3/4 & 1/4 \end{pmatrix}$

**20** $X \sim Unif(-1,1)$

$f_X(x) = \begin{cases} 0, & x<-1 \\ \frac{1}{2}, & -1 \le x \le 1 \\ 0, & x>1 \end{cases}$ $\quad F_X(x) = \begin{cases} 0, & x<-1 \\ \frac{x+1}{2}, & -1 \le x < 1 \\ 1, & x>1 \end{cases}$

$Y = |X|$

$\underset{\uparrow \text{ параметар}}{F_Y(x)} = P(Y \le x) = P(|X| \le x) = \begin{cases} 0, & x \le 0 \\ P(-x \le X \le x), & x>0 \end{cases}$

$P(-x \le X \le x) = F_X(x) - F_X(-x) = \begin{cases} \frac{x+1}{2} - \frac{1-x}{2}, & 0 \le x \le 1 \\ 1, & x>1 \end{cases}$

$F_Y(x) = \begin{cases} 0, & x<0 \\ x, & 0 \le x \le 1 \\ 1, & x>1 \end{cases}$ $\quad F_Y'(x) = f_Y(x) = \begin{cases} 0, & x<0 \\ 1, & 0 \le x \le 1 \\ 0, & x>1 \end{cases} \sim Unif(0,1)$

**17**

| x \ y | −1 | 0 | 1 | |
|---|---|---|---|---|
| −1 | 1/8 | 0 | c | 3/8 |
| 1 | 1/2 | 1/4 | 0 | 6/8 |
| | 5/8 | 1/4 | 1/8 | |

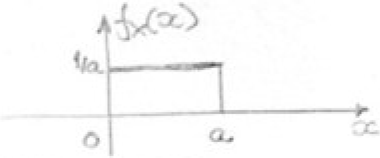
$\frac{1}{2}+\frac{1}{8}$

$$1/8 + 0 + c + \overset{\cdot 4}{1/2} + \overset{\cdot 2}{1/4} + 0 = 1$$

$$\frac{7}{8} + c = 1 \Rightarrow \boxed{c = \frac{1}{8}}$$

$$X : \begin{pmatrix} -1 & 1 \\ 1/4 & 3/4 \end{pmatrix} \qquad Y : \begin{pmatrix} -1 & 0 & 1 \\ 5/8 & 2/8 & 1/8 \end{pmatrix}$$

**18** $X \sim Unif(0, a)$ $\quad P(X < 1{,}5) = 3/4$




$$f_X(x) = \begin{cases} 0, & x < 0 \\ \frac{1}{a}, & 0 \le x \le a \\ 0, & x > a \end{cases} \qquad F_X(x) = \begin{cases} 0, & x < 0 \\ \frac{x}{a}, & 0 \le x \le a \\ 1, & x > a \end{cases}$$

$\boxed{\text{тч} : a > 0}$

$$P(X < 1{,}5) = F_X(1{,}5) = \int_{-\infty}^{1{,}5} f_X(x)\,dx = \frac{3}{4} \Rightarrow a > 1{,}5$$

$$\int_0^{1{,}5} \frac{1}{a}\,dx = \frac{3}{4} \qquad \frac{x}{a}\Big|_0^{1{,}5} = \frac{3}{4} \qquad \frac{3}{2a} = \frac{3}{4} \qquad \boxed{a = 2}$$

**19** $X : \begin{pmatrix} -2 & 2 & 5 \\ 1/4 & 1/2 & c \end{pmatrix}$ $\quad 1/4 + 1/2 + c = 1 \Rightarrow \boxed{c = \frac{1}{4}}$

$$Y = X^2 \qquad Y \in \{4, 25\} \qquad Y = \begin{pmatrix} 4 & 25 \\ 1/4 + 1/2 & 1/4 \end{pmatrix} = \begin{pmatrix} 4 & 25 \\ 3/4 & 1/4 \end{pmatrix}$$

**20** $X \sim Unif(-1, 1)$

$$f(x) = \begin{cases} 0, & x < -1 \\ \frac{1}{2}, & -1 \le x \le 1 \\ 0, & x > 1 \end{cases} \qquad F_X(x) = \begin{cases} 0, & x < -1 \\ \frac{x+1}{2}, & -1 \le x \le 1 \\ 1, & x > 1 \end{cases}$$

$Y = |X|$

$$\underset{\substack{\uparrow \\ \text{параметар}}}{F_Y(x)} = P(Y \le x) = P(|X| \le x) = \begin{cases} 0, & x \le 0 \\ P(-x \le X \le x), & x > 0 \end{cases}$$

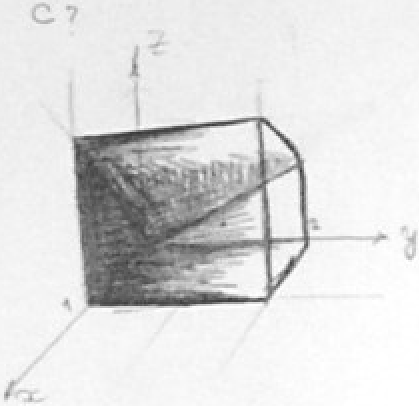
$$P(-x \le X \le x) = F_X(x) - F_X(-x) = \begin{cases} \frac{x+1}{2} - \frac{1-x}{2}, & 0 \le x \le 1 \\ 1, & x > 1 \end{cases}$$

$$F_Y(x) = \begin{cases} 0, & x < 0 \\ x, & 0 \le x \le 1 \\ 1, & x > 1 \end{cases} \qquad F_Y'(x) = f_Y(x) = \begin{cases} 0, & x < 0 \\ 1, & 0 \le x \le 1 \\ 0, & x > 1 \end{cases} \sim Unif(0, 1)$$

21

$$f_{X,Y}(x,y)=\begin{cases} c(2x+y), & 0<x<1,\ 0<y<2 \\ 0, & \text{иначе} \end{cases}$$

c ?




$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f_{XY}(x,y)\,dx\,dy = 1$$

$$c\int_0^1 dx\int_0^2 (2x+y)\,dy = 1$$

$$c\int_0^1 \left(2xy+\frac{y^2}{2}\right)\Big|_0^2 dx = 1$$

$$c\int_0^1 (4x+2)\,dx = 1$$

$$\left(c\cdot 4\frac{x^2}{2}+c\cdot 2x\right)\Big|_0^1 = 1$$

$$2c+2c=1 \qquad \boxed{c=1/4}$$

маргинална густина расподеле за X

$$f_X(x)=\int_{-\infty}^{+\infty} f_{X,Y}(x,y)\,dy = \int_0^2 \frac{1}{4}(2x+y)\,dy = \frac{1}{4}\left(2xy+\frac{y^2}{2}\right)\Big|_0^2 =$$

$$=\frac{1}{4}(4x+2)=\boxed{x+\frac{1}{2}}$$

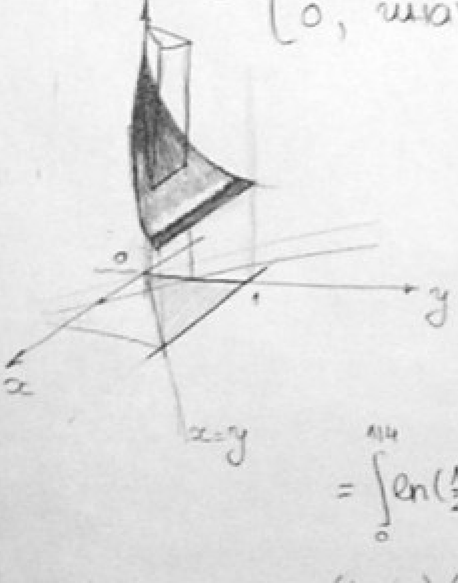
маргинална густина расподеле за Y

$$f_Y(y)=\int_{-\infty}^{+\infty} f_{X,Y}(x,y)\,dx = \int_0^1 \frac{1}{4}(2x+y)\,dx = \frac{1}{4}(x^2+xy)\Big|_0^1 =$$

$$=\boxed{\frac{1}{4}(y+1)}$$

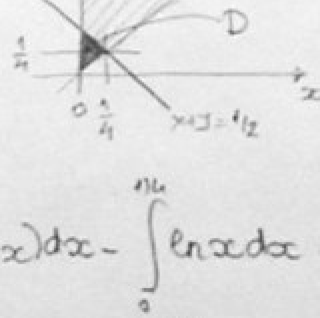
22

$$f_{X,Y}(x,y)=\begin{cases} 1/y, & 0<x<y,\ 0<y<1 \\ 0, & \text{иначе} \end{cases}$$




$x+y=1/2$

$y=1/2-x$




$$P(X+Y\le 1/2)=\iint_D f_{X,Y}(x,y)\,dx\,dy =$$

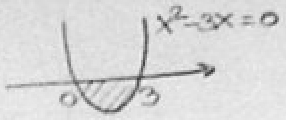
$$=\int_0^{1/4} dx\int_x^{1/2-x}\frac{1}{y}\,dy =$$

$$=\int_0^{1/4} \ln|y|\Big|_x^{1/2-x} dx =$$

$$=\int_0^{1/4}\ln\left(\tfrac{1}{2}-x\right)dx-\int_0^{1/4}\ln x\,dx =$$

$$=-\left(\tfrac{1}{2}-x\right)\left(\ln\left(\tfrac{1}{2}-x\right)-1\right)\Big|_0^{1/4}-x(\ln x-1)\Big|_0^{1/4}=\boxed{\frac{\ln 2}{2}}$$

**23** $X \sim N(2,4)$ $\quad \mu \quad \sigma^2$




$Z \sim N(0,1)$

$P(X^2-3X \leq 0) =$

$= P(0 \leq X \leq 3) =$ $\qquad Z = \dfrac{X-2}{2}$ $\qquad \boxed{Z = \dfrac{X-\mu}{\sigma}}$

$= P(-2 \leq X-2 \leq 1) = P\left(-1 \leq \frac{X-2}{4} \leq \frac{1}{2}\right) = F_Z\left(\frac{1}{2}\right) - F_Z(-1) =$

$= \Phi(0{,}5) - 1 + \Phi(1) = 0{,}6915 - 1 + 0{,}8413 = 0{,}5328$

**24** $X \sim N(m, 25)$ $\quad P(X > 7) = 0{,}1151$

одредити EX – математичко очекивање

$EX = m$

$Z = \dfrac{X-m}{5}$ $\qquad P(X-m > 7-m) = 0{,}1151$

$P\left(\frac{X-m}{5} > \frac{7-m}{5}\right) = 0{,}1151 = 1 - P\left(Z < \frac{7-m}{5}\right)$

$\Phi\left(\frac{7-m}{5}\right) = 0{,}8849$

$\Downarrow$

$\dfrac{7-m}{5} = 1{,}2$

$7 - m = 6$

$\boxed{m = 1}$

**25** $X \sim N(-1, \sigma^2)$

$P(X \leq 2{,}46) = 0{,}97725$ $\qquad \mathrm{Var}X = \sigma^2$

$Z = \dfrac{X+1}{\sigma}$

$P(X+1 \leq 3{,}46) = P(X \leq 2{,}46)$

$P\left(\frac{X+1}{\sigma} \leq \frac{3{,}46}{\sigma}\right) = 0{,}97725$

$\dfrac{3{,}46}{\sigma} \approx 2$

$\sigma \approx 1{,}73$ $\qquad \boxed{\mathrm{Var}X = 3}$

**26** $X \sim N(0,2)$ $\qquad Z = 2X - 3Y$

$Y \sim Exp(2)$

$EZ, \mathrm{Var}Z$ $\qquad \boxed{E(Z)} = E(2X-3Y) = 2E(X) - 3E(Y) = \boxed{-\frac{3}{2}}$

$E(X) = 0$ $\qquad \mathrm{var}(X) = \sigma^2 = 2$

$EY = 2\int_{-\infty}^{+\infty} x e^{-2x} dx = 2\int_0^{+\infty} x e^{-2x} dx = \frac{1}{2}\Gamma(2) = \frac{1}{2}$

$\mathrm{Var}(Y) = E(Y^2) - EY^2 =$
$= 2\int_{-\infty} x^2 e^{-2x} dx - \frac{1}{2}\int_0 4x^2 e^{-2x} dx =$
$= \frac{1}{4}\Gamma(3)$
$\downarrow$
$1/2$

$\mathrm{Var}(2X-3Y) = \mathrm{Var}(2X + (-3)Y) = \mathrm{Var}(2X) + \mathrm{Var}((-3)Y) = 2^2 \mathrm{Var}(X) + (-3)^2 \mathrm{Var}(Y)$

$= 4\mathrm{Var}(X) + 9\mathrm{Var}(Y) = 8 + \frac{9}{2} = \boxed{\frac{25}{2}}$

**27**

| $X \backslash Y$ | 0 | 1 | 2 | |
|---|---|---|---|---|
| 1 | 0 | 1/12 | 1/4 | 2/6 |
| 2 | 1/6 | 1/2 | 0 | 4/6 |
| | 2/12 | 5/12 | 3/12 | |

$$Y: \begin{pmatrix} 0 & 1 & 2 \\ 2/12 & 5/12 & 3/12 \end{pmatrix}$$

$$X: \begin{pmatrix} 1 & 2 \\ 2/6 & 4/6 \end{pmatrix}$$

$X+Y$

$1+0=1 \qquad 2+0=2$

$1+1=2 \qquad 2+1=3$

$1+2=3 \qquad 2+2=4$

$$X+Y: \begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & \frac{1}{12}+\frac{1}{6}^{\llcorner 2} & \frac{1}{4}^{\llcorner 3}+\frac{1}{2}^{\llcorner 6} & 0 \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ \frac{3}{12} & \frac{9}{12} \end{pmatrix}$$

$$E(X+Y) = \sum p_i (x+y)_i = \frac{\cancel{3}}{\underset{4}{\cancel{12}}}\cdot 2 + \frac{9}{\underset{4}{\cancel{12}}}\cdot \cancel{3} = \boxed{\frac{11}{4}}$$

**28**

| $X \backslash Y$ | 0 | 1 | |
|---|---|---|---|
| 1 | 1/6 | 1/4 | 5/12 |
| 2 | 1/3 | 1/4 | 7/12 |
| | 1/2 | 1/2 | |

$P(X=a, Y=b) = P(X=a)P(Y=b) \quad \forall x, y$

$P(X=1, Y=0) = \frac{1}{6} \quad P(X=1)\cdot P(Y=0) = \frac{5}{12}\cdot\frac{1}{2} \neq \frac{1}{6}$

нису независне!

$$\mathrm{Cov}(X,Y) = E((X-E(X))(Y-E(Y)))$$

$$E(X) = 1\cdot\frac{5}{12} + 2\cdot\frac{7}{12} = \frac{19}{12}$$

$$E(Y) = 0\cdot\frac{1}{2} + 1\cdot\frac{1}{2} = \frac{1}{2}$$

$$E\left(\left(X-\frac{19}{12}\right)\left(Y-\frac{1}{2}\right)\right) = E\left(XY - \frac{19}{12}Y - \frac{1}{2}X + \frac{19}{24}\right)$$

$$\mathrm{Cov}(X,Y) = E(XY) - E(X)E(Y) = E(XY) - \frac{19}{24} = \frac{3}{4}^{\llcorner 6} - \frac{19}{24} = \boxed{-\frac{1}{24}}$$

$XY$:

$1\cdot 0 = 0 \qquad 2\cdot 0 = 0$

$1\cdot 1 = 1 \qquad 2\cdot 1 = 2$

$$XY: \begin{pmatrix} 0 & 1 & 2 \\ \frac{1}{6}+\frac{1}{3} & 1/4 & \frac{1}{4} \end{pmatrix} = \begin{pmatrix} 0 & 1 & 2 \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \end{pmatrix}$$

$$E(XY) = 0\cdot\frac{1}{2} + 1\cdot\frac{1}{4} + 2\cdot\frac{1}{4} = \frac{3}{4}$$

**29**

$$f_X(x) = \begin{cases} x &, 0 < x < 1 \\ 2-x &, 1 \le x < 2 \\ 0 &, \text{иначе} \end{cases} \qquad EX\,?$$

$$\boxed{EX} = \int_{-\infty}^{+\infty} x f_X(x)\,dx = \int_0^1 x\cdot x\,dx + \int_1^2 x(2-x)\,dx =$$

$$= \frac{x^3}{3}\Big|_0^1 + x^2\Big|_1^2 - \frac{x^3}{3}\Big|_1^2 = \frac{1}{3} + 4 - 1 - \frac{8}{3} + \frac{1}{3} = \boxed{1}$$

$\boxed{30}$ $X \sim Unif(a,b)$

$$f_X(x) = \begin{cases} \frac{1}{b-a}, & a \le x \le b \\ 0, & \text{иначе} \end{cases} \qquad b > a$$

$$EX = \int_{-\infty}^{+\infty} x f_X(x)\,dx = \int_a^b \frac{x}{b-a}\,dx = \frac{x^2}{2(b-a)}\Big|_a^b = \frac{b^2-a^2}{2(b-a)} = \frac{a+b}{2}$$

$$\boxed{a+b=16} \quad (1)$$

$$Var(X) = E(X^2) - EX^2$$

$$E(X^2) = \int_{-\infty}^{+\infty} x^2 f_X(x)\,dx = \int_a^b \frac{x^2}{b-a}\,dx = \frac{x^3}{3(b-a)}\Big|_a^b = \frac{b^2+ab+a^2}{3}$$

$$\boxed{\frac{4}{3} = \frac{b^2+ab+a^2}{3} - 64} \quad (2)$$

$$4 = b^2+ab+a^2 - 192$$

$$b^2+ab+a^2 = 196$$

$$(16-a)^2 + 16a - \cancel{a^2} + \cancel{a^2} = 196$$

$$256 + a^2 - 32a + 16a = 196$$

$$a^2 - 16a + 60 = 0$$

$$(a-6)(a-10) = 0$$

| $a_1 = 6$ | $b_1 = 10$ |
|---|---|
| $a_2 = 10$ | $b_2 = 6$ |